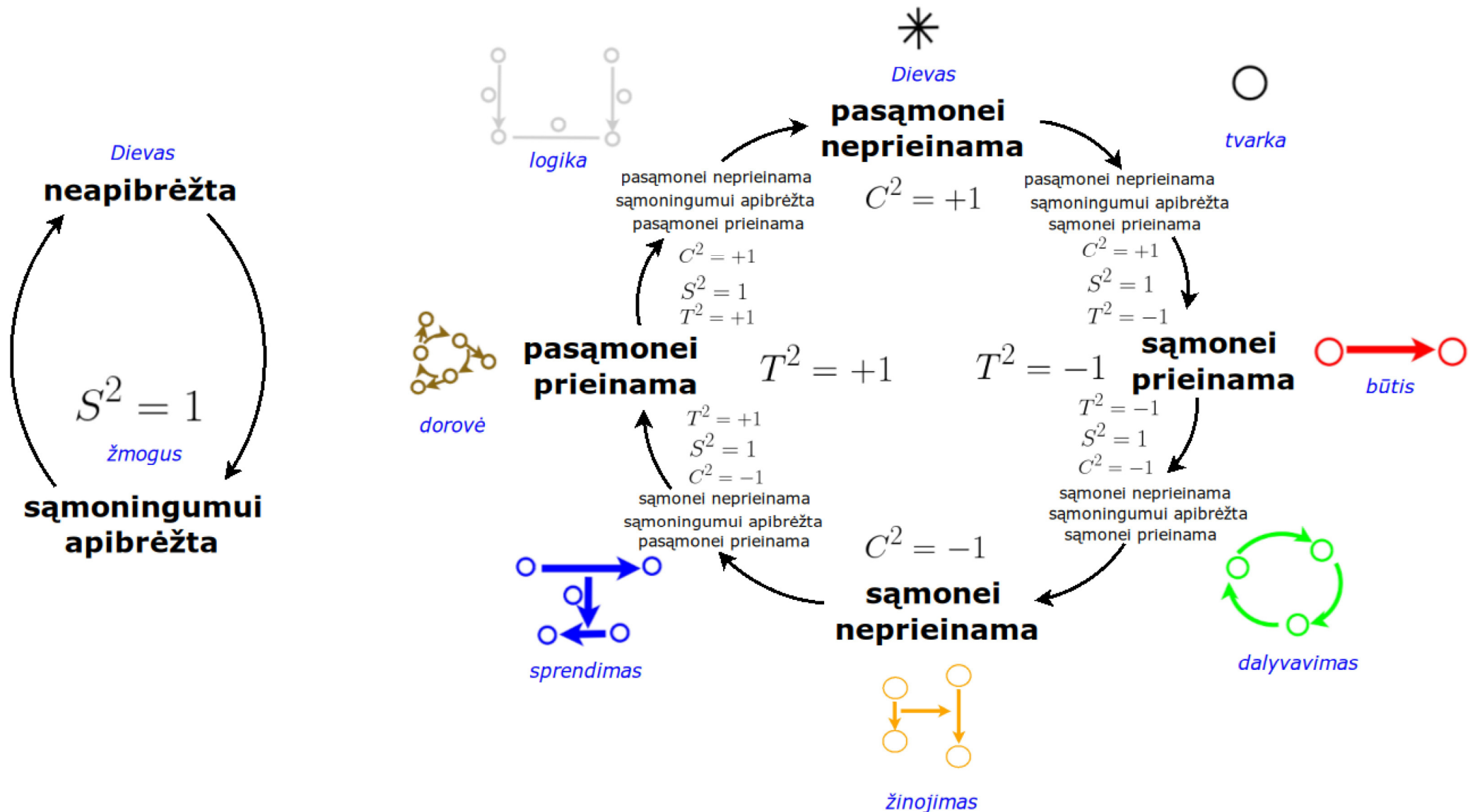


Savasties būsenų modeliavimas

hamiltoniano 3 kvantinėmis simetrijomis bei Bott periodiškumu



3 protai

|一

pašmonė
žino, sako
prieinama

|二

sąmonė
nežino, klausia
neprieinama

|三

sąmoningumas
lygina, tiria
apibrėžta

3 transformacijos

laiko
apsukimas

T
 $(x, -t)$

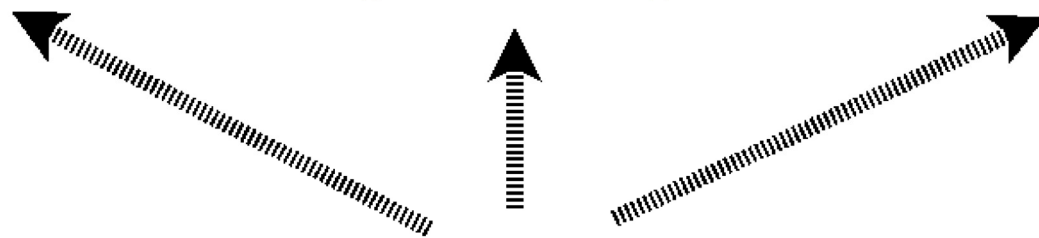
krūvio
sukeitimas

C
 $(-x, -t)$

veidrodinis
atspindys

S
 $(-x, t)$

(x, t)

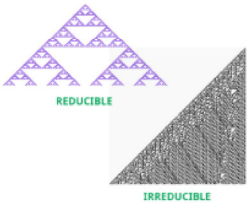
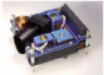
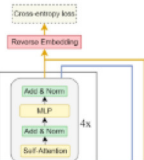


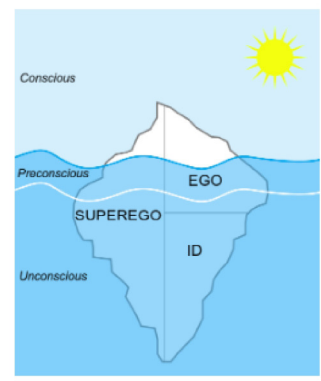


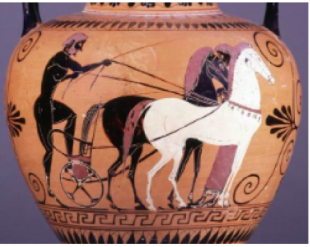
Theory Translator is powered by [Wondrous Wisdom](#) and curated by [Andrius Kulikauskas](#).
[View all records](#), [new suggestions](#), [latest entries](#). Illustrating 9 conceptual structures with 511 examples. [Suggest an example!](#)

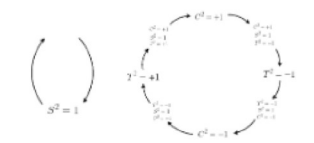
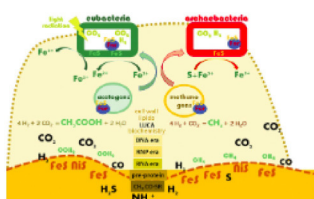
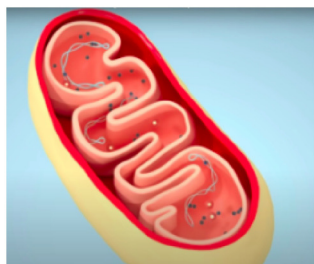
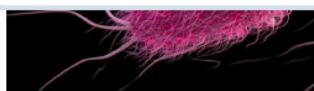
Think structurally! Select a conceptual structure below to investigate:
[Three minds acting on the nullsome](#), [onesome](#), [twosome](#), [threesome](#), [foursome](#), [fivesome](#), [sixsome](#), [sevensome](#).

three minds

	answering	questioning	investigating	Showing 354 examples. Read an article about the three minds .	~ year
	irreducibility	reducibility		British-American computer scientist Stephen Wolfram proposed that the arrow of time is a consequence of computational reducibility. ... one of the questions in economics is what is value? What makes something valuable? ... computational reducibility. ... in the world at large, there's lots of computational irreducibility, lots of things that are unpredictable, lots of things you can't do quickly, and so on. But we humans have one thing in fairly short supply, and that's time ... So for us, anything that kind of speeds up what we can achieve is something that is valuable to us. And computational reducibility is the possibility of finding these little pockets where you can kind of jump ahead ... Stephen Wolfram. The Simple Reason for Time's Arrow That Physics Missed. #1684	2025
 ADCS (Attitude Determination and Control System) An ADCS is a sub-system that determines and maintains a spacecraft's orientation in space.	agents	agents and instruments		Italian philosopher Luca Possati appealed to Active Inference to bridge two conceptions of agency. Things go far beyond merely affecting what we do - they change and impact how reality is disclosed to us. a) the strong view, exemplified by Latour and ANT [Actor-network theory], which develops a symmetrical or "flat" ontology in which agency is distributed equally among humans and nonhumans, with no ontological or epistemological distinction between different types of agency; b) the weak view, represented by postphenomenology, the work of ecological psychologists, classical phenomenology, and anthropology, which maintains an ontological and epistemological distinction between human agency and nonhuman agency. Luca Possati. What is an artifact? Active inference and technology. #1683	2025
				Canadian Artificial Intelligence researcher Alexia Jolicoeur-Martineau developed Tiny Recursive Model, a recursive reasoning approach with only 2 layers and 7 million parameters, which scored 45% on the ARC-AGI-1 test, higher than many Large Language Models. Figure 1. Tiny Recursion Model (TRM) recursively improves its predicted answer y with a tiny network. It starts with the embedded input question x and initial embedded answer y, and latent z. For up to $N_{sup} = 16$ improvements steps, it tries to improve its answer v. It does so by	

$[\bar{p}, \bar{\xi}, N(\bar{\xi})]$	facts	thoughts	truth	Austrian philosopher Ludwig Wittgenstein, in his Tractatus Logico-Philosophicus, elucidated seven theses comprising clear thinking. <i>The book deals with the problems of philosophy and shows, as I believe, that the method of formulating these problems rests on the misunderstanding of the logic of our language. Its whole meaning could be summed up somewhat as follows: What can be said at all can be said clearly; and whereof one cannot speak thereof one must be silent.</i> 1. The world is all that is the case. 2. What is the case—a fact—is the existence of states of affairs. 3. A logical picture of facts is a thought. 4. A thought is a proposition with sense. 5. A proposition is a truth-function of elementary propositions. (An elementary proposition is a truth function of itself.) 6. The general form of a truth-function is $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$ This is the general form of a proposition. 7. Whereof one cannot speak, thereof one must be silent. What we cannot speak about we must pass over in silence. Stanford. Ludwig Wittgenstein. Ludwig Wittgenstein. Tractatus Logico-Philosophicus. #902 📌	1921
	id	superego	ego	Jewish Austrian neurologist Sigmund Freud, founder of psychoanalysis, introduced id, ego, superego in his 1920 essay "Beyond the Pleasure Principle" and formalized them in his 1923 essay "The Ego and the Id". - The id is the dark, inaccessible part of our personality, the unconscious source of bodily needs and wants, emotional impulses and desires, satisfying drives (aggressive and sexual) by seeking pleasure. - The ego is the reason and common sense, the organizing principle for thoughts and interpretations, which minds the reality of the external world, seeks self-preservation by balancing the demands of the id, external world and superego. - The superego is the conscience, the internalization of cultural rules, absorbed from parents and authorities, expressing ideals and personal goals. Wikipedia: Id, ego and superego #4 📌	1920
	imaginative	descriptive	adaptive	Scottish literary critic George Gregory Smith described Scottish literature as being of two moods, restless, never lining up, a Caledonian antisyzygy, a zigzag of contradictions, combining opposites - Saxon descriptiveness and Celtic imaginativeness, yielding adaptiveness, practical judgement. <i>So it has fallen out with Scottish literature, as Burns tells us it fell out with his art, that it has been cared for by fairies, and brownies, and witches, and warlocks, and spunkies, and kelpies - "and other trumpery," he adds, with a Scot's privilege and a Scottish meaning. So, too, this literature has satisfied that "first condition of the poetic way of seeing and presenting things" which is, in Pater's phrase, "particularisation" and "the delight in concrete definition." Contraries indeed, but as warp and woof.</i> G.Gregory Smith. Scottish literature, character & influence. Wikipedia: Caledonian Antisyzygy #1553 ❤️ Marcus Petz 📌	1919

plato	< > 7 of 16		rhetoric	logic	dialectic	Aristotle, in his Rhetoric, distinguishes between rhetoric and dialectic. He also was a founder of classical logic, although the term "logic" came later. All three have the function of providing arguments. We can infer a distinction. • Rhetoric considers what arguments are persuasive or not. • Logic considers what arguments are, of themselves, valid or not. • Dialectic considers what arguments are illuminating in arguing for and against. Stanford: Aristotle's Rhetoric #124	-357
			pathos	logos	ethos	Greek philosopher Aristotle, in his Rhetoric, distinguished three modes of persuasion - grounds on which speakers appeal to their audiences: • pathos (suffering or experience) evokes emotions, appeals to what is familiar to the audience, their ideals • ethos (character) appeals to the speaker's authority and credibility, their qualifications, their character, how they balance passion and caution • logos (word, discourse or reason) is an appeal to logic, facts and figures, derivations Wikipedia: Modes of persuasion #97	-357
			irrational appetite	moral impulse	reason	Greek philosopher Plato, in the Phaedrus, presents the Allegory of the Chariot. The charioteer drives a chariot pulled by two winged horses. <i>First the charioteer of the human soul drives a pair; and secondly one of the horses is noble and of noble breed, but the other quite the opposite in breed and character. Therefore in our case the driving is necessarily difficult and troublesome.</i> The charioteer represents reason, which guides the soul to truth. The noble horse is the positive passion, the rational and moral impulse. The ignoble horse is the negative, irrational passion, the appetites. Wikipedia: Phaedrus (dialogue) #121	-370
			appetite	spunk	reason	Greek philosopher Plato described in his Republic three classes which manifested three parts of the soul. • Commoners (farmers, artisans) should live in moderation. They manifest appetite (epithumetikon). • Warriors should moreover be brave. They manifest spunk (thumetikon). • Rulers should moreover be wise. They manifest reason (logistikon). Wikipedia: Plato's theory of soul #5	-375
			think one knows	know nothing	be conscious	In Plato's Apology, Socrates, at his trial in Athens for impiety and corrupting the youth, said <i>...although I do not suppose that either of us knows anything really beautiful and good, I am better off than he is - for he knows nothing, and thinks that he knows. I neither know nor think that I know.</i>	-399



randomness	gradient	computation
cell environment	communication	communal health
biofilm	cells	community
time reversal	charge conjugation	parity

swim up the gradient to get food and stuff like that. ... in the absence of a food gradient, what these things do is they swim around according to a path that has power law statistics. This is their tumbling. The random tumbling thing. ... So in a sense, their behavior is scale invariant and the computation that their simple little body is doing is actually another example of criticality.

[Brain Inspired. Woodrow Shew and Keith Hengen: The Nature of Brain Criticality.](#) #1468

Russian Canadian podcaster Anton Petrov summarized research findings about mitochondria. So this powerhouse analogy while catchy may not be entirely correct. Mitochondria are way more complex and one of their main functions is actually information processing inside the cell. But as we've discussed before, not just inside one cell, they can even interact and process information outside of the cell while then communicating with each other. And that's because they seem to be also equipped with a lot of different receptors on their surface that can easily sense what's happening inside a cell, can easily receive certain molecules used for communication, and then integrate all of this information. And here's the kicker. Then tell the nucleus inside the cell on what to do. Basically, mitochondria direct nucleus and other organelles in order to maintain the health of the cell and thus the health of the entire organism. So I guess here you can also see them as a kind of a brain or the mind of the cell itself. They seem to be controlling the whole show. And so a lot of the recent research describes mitochondria as dynamic sensing organelles able to perceive, able to integrate information and then to adapt in order to provide thriving conditions for the cell. Which of course does actually make sense because we know that billions of years ago this used to be an independent bacteria. So they very likely maintain a lot of functions and can still act independently, although in this case also seem to act as a kind of a communal organism.

[Anton Petrov. Research on Mitochondria Provides Answers on Why We Have to Sleep.](#)
[Martin Picard, Carmen Sandi. The social nature of mitochondria: Implications for human health.](#) #1508

Swedish medical microbial physiologist Ute Römling argued that biofilms arose along with the most ancient life. As previously hypothesized in this opinion paper, biofilms originated concomitantly with life. The most fundamental behaviour towards ancient solid electron donors and acceptors and the cellular prerequisites for direct most effective energy gain from solid mineral electron acceptors was therefore biofilm formation. With most ancient and most abundant iron and sulfur-based respiration requiring solid surfaces, biofilm formation could be subsequently extended towards additional solid inorganic electron acceptors. Upon the availability of soluble inorganic and organic electron acceptors and other energy-related signals, solid-surface biofilm formation desisted to be mandatory for energy-gain, but a multicellular microbial community in the form of cell aggregates (flocks) has still been a prerequisite for survival as even today, single planktonic microbial cells are rarely found in nature. With the presence of solid and soluble redox pairs, the biofilm response could subsequently further diversify.

[Ute Römling. Is biofilm formation intrinsic to the origin of life?](#) #1435

The universe is invariant upon applying together all three fundamental symmetry operations: charge conjugation C (swapping matter and antimatter), parity P (reflecting in a mirror, swapping lefthandedness and righthandedness), and time reversal T (changing the direction in time, swapping forwards and backwards).

[Wikipedia: CPT symmetry.](#)
[Kulikauskas. An Allegory: The Solipsistic Self as the Hamiltonian of a Noninteracting Fermion.](#) #1310

-800000000
-1850000000
-3650000000
-13800000000

MODELIAVIMAS YRA TILTAS

išmintis

savęs pažinimas
vaizduotės ribų pažinimas
išgyvenimų nusakymas



metafizika

psichologija

kultūra

neurologija

evoliucija



matematika

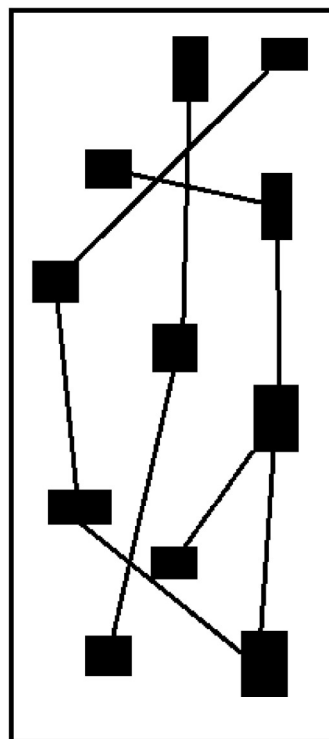
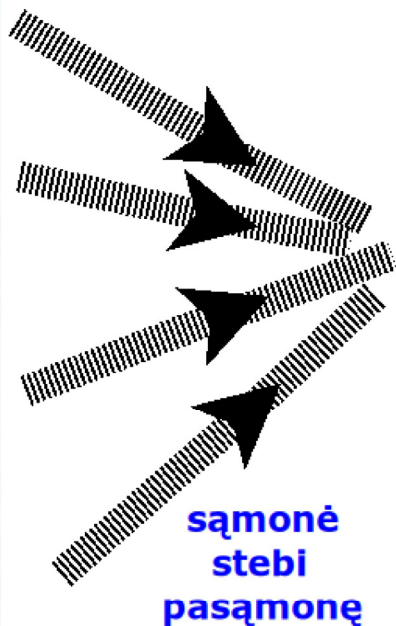
matematinis mąstymas



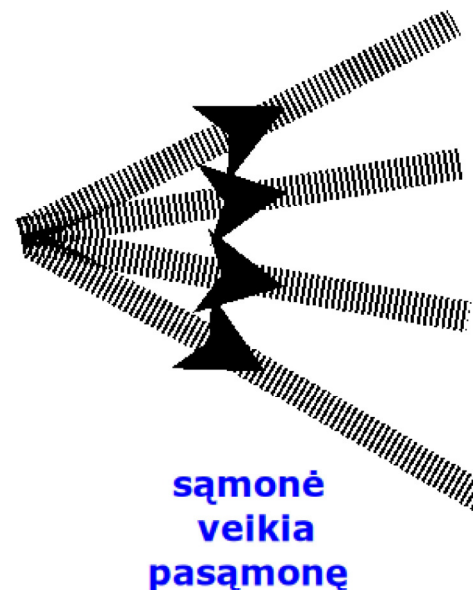
fizika



suglaudžia



išskleidžia



100,000,000,000
neuronų
pasąmonė

100,000
sąvokų
sąmonė

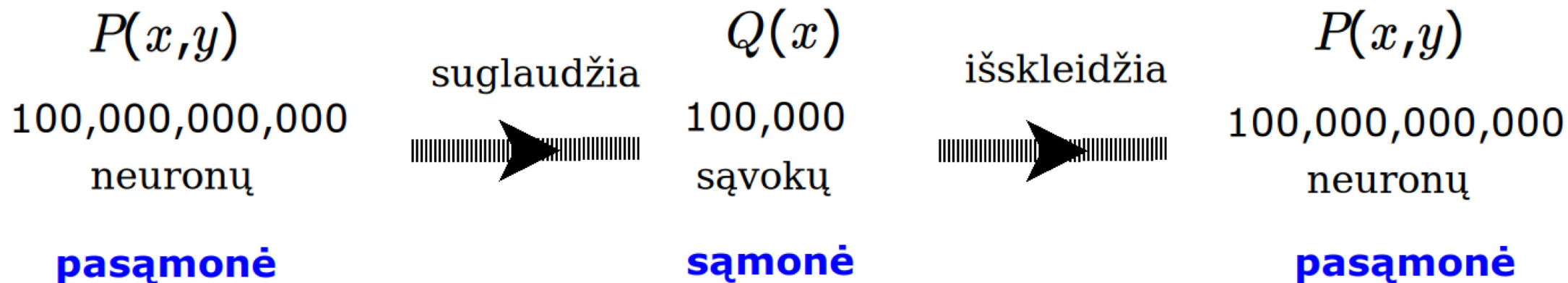
100,000,000,000
neuronų
pasąmonė



sulygina
sąmoningumas

verčia sąmonę neskubėti

vertina sąmonės atliktą darbą



$$\frac{1}{P(x,y)} = \frac{Q(x)}{P(x,y)} \frac{1}{Q(x)}$$

$$\ln \frac{1}{P(x,y)} = \ln \frac{1}{\frac{P(x,y)}{Q(x)}} + \ln \frac{1}{Q(x)}$$

$$\sum_x Q(x) \ln \frac{1}{P(x,y)} = \sum_x Q(x) \ln \frac{1}{\frac{P(x,y)}{Q(x)}} + \sum_x Q(x) \ln \frac{1}{Q(x)}$$

$$\frac{\text{energija}}{\text{temperatūra}} = \frac{\text{laisvoji energija}}{\text{temperatūra}} + \text{entropija}$$

$$\frac{Q(x)}{P(x,y)} = \frac{Q(x)}{P(x|y)} \frac{1}{P(y)}$$

$$\ln \frac{1}{\frac{P(x,y)}{Q(x)}} = \ln \frac{1}{\frac{P(x|y)}{Q(x)}} + \ln \frac{1}{P(y)}$$

$$\sum_x Q(x) \ln \frac{1}{\frac{P(x,y)}{Q(x)}} = \sum_x Q(x) \ln \frac{1}{\frac{P(x|y)}{Q(x)}} + \sum_x Q(x) \ln \frac{1}{P(y)}$$

$$\frac{\text{laisvoji energija}}{\text{temperatūra}} = \text{neatitikimas} + \text{sužinojimas}$$

$$E \pm \varepsilon_E$$

NUMANYMAS
SU PAKLAIDA



$$M \pm \varepsilon_M$$

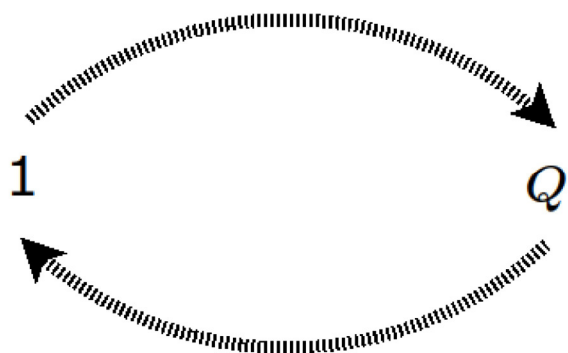
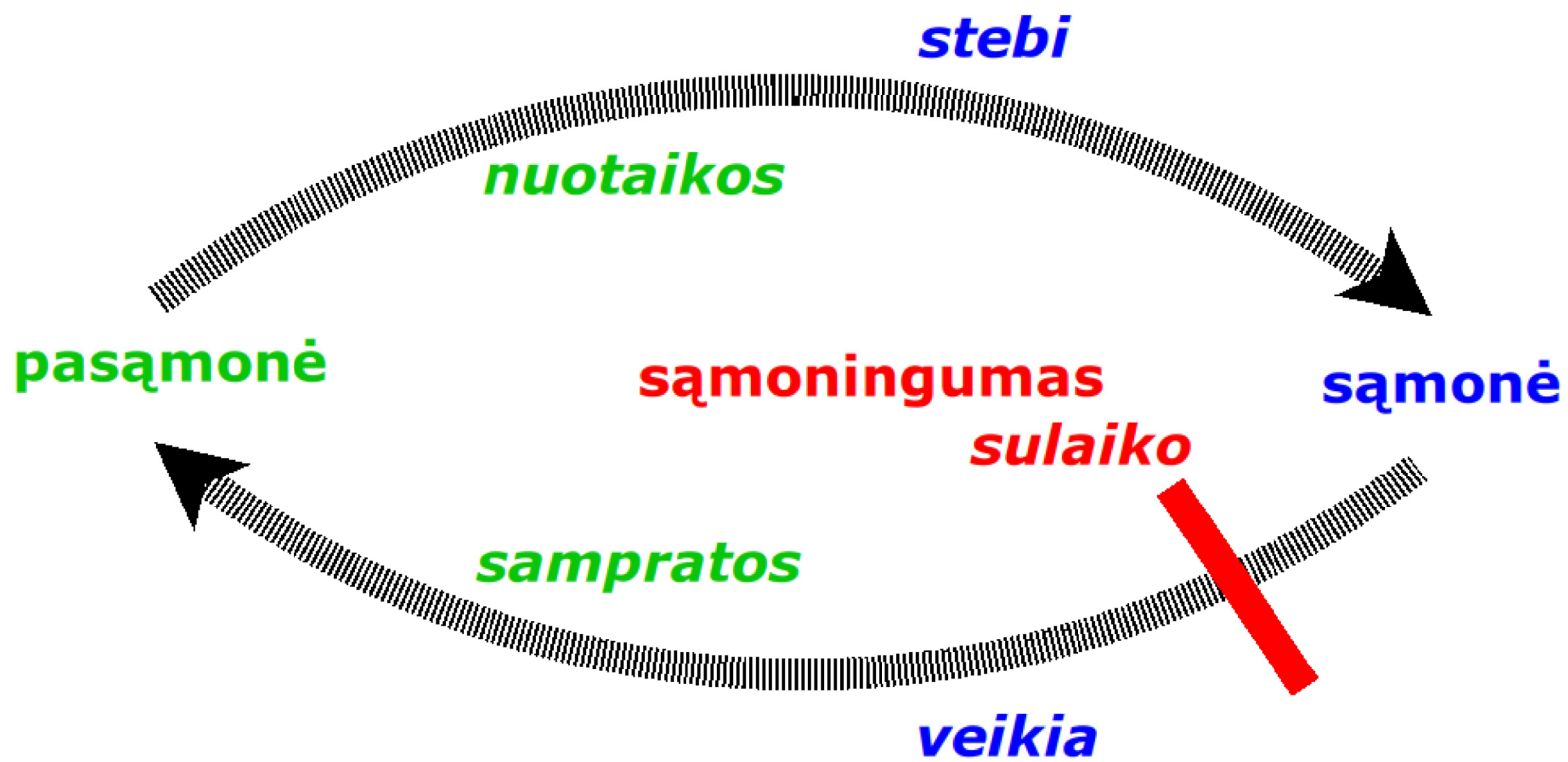
MATAVIMAS
SU PAKLAIDA



KALMANO
FILTRAS

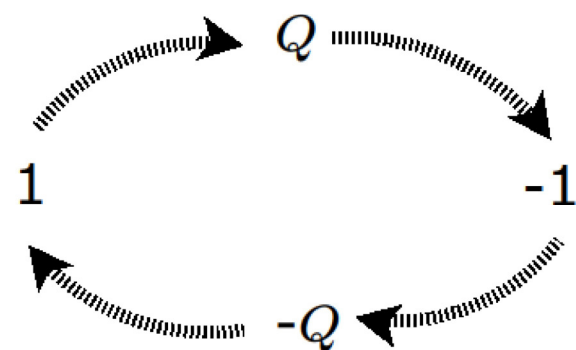
$$\frac{E\varepsilon_M + M\varepsilon_E \pm \varepsilon_M\varepsilon_E}{\varepsilon_E + \varepsilon_M}$$

ATNAUJINTAS
NUMANYMAS
SU PAKLAIDA

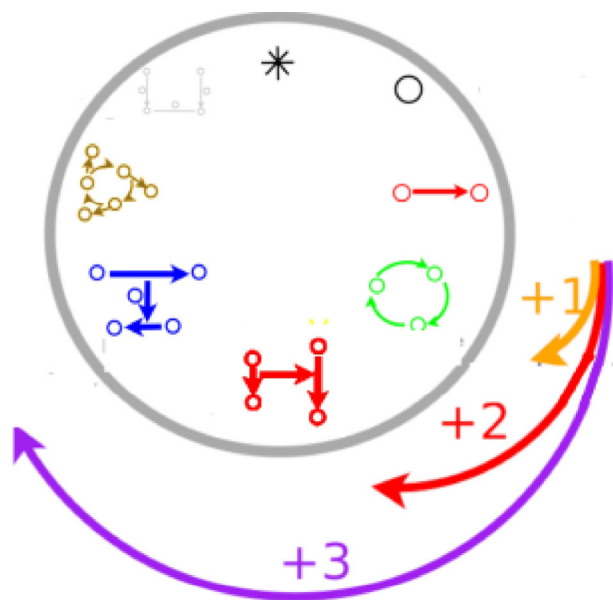


$Q^2 = 1$ **pasąmonė**

$Q^4 = 1$

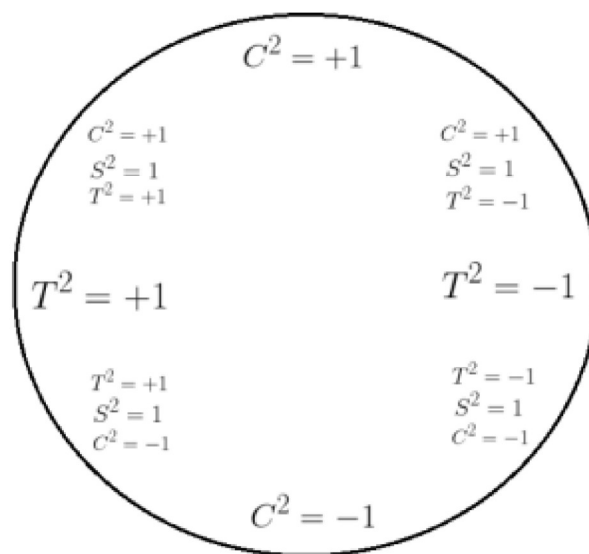


sąmonė $Q^2 = -1$



visko padalinimai

Kulikauskas 1989

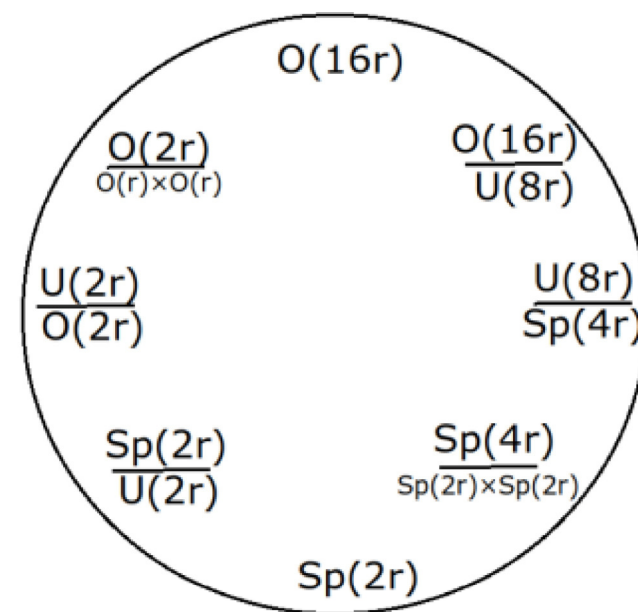


kvantinių simetrijų rinkiniai

Altland, Zirnbauer 1996

Kitaev 2009

Agarwala, Haldar, Shenoy 2016



simetrinės erdvės (Hamiltonianai)

Cartan 1926

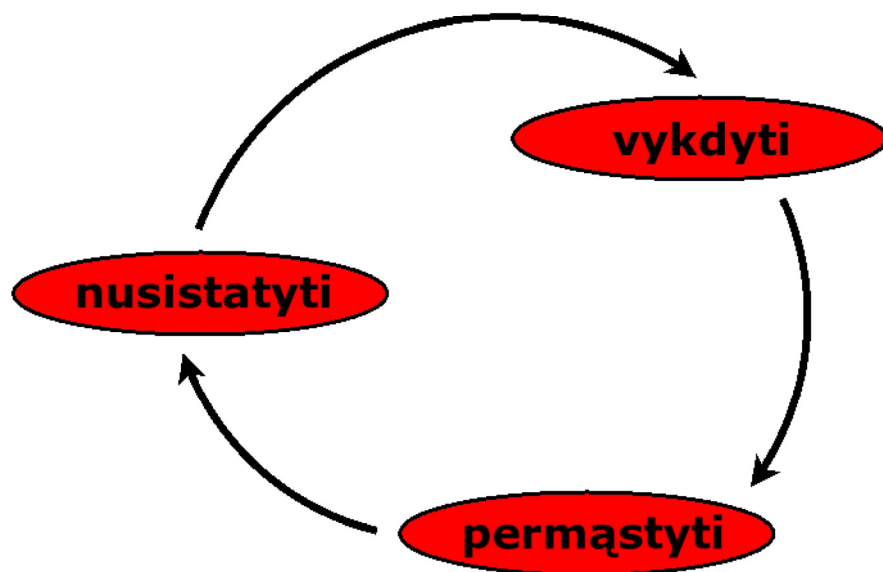
Bott 1957

Stone, Chiu, Roy 2010

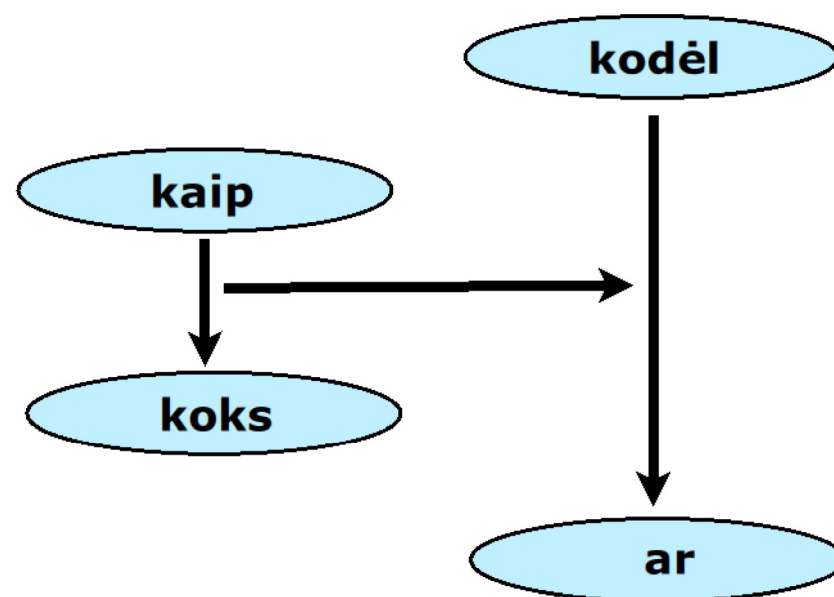
BŪTIS



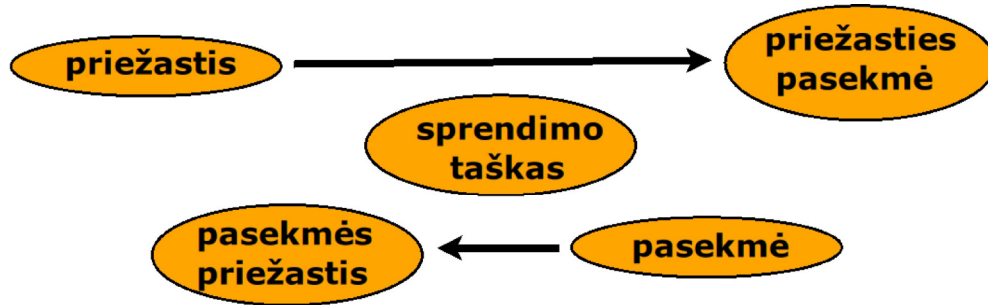
DALYVAVIMAS



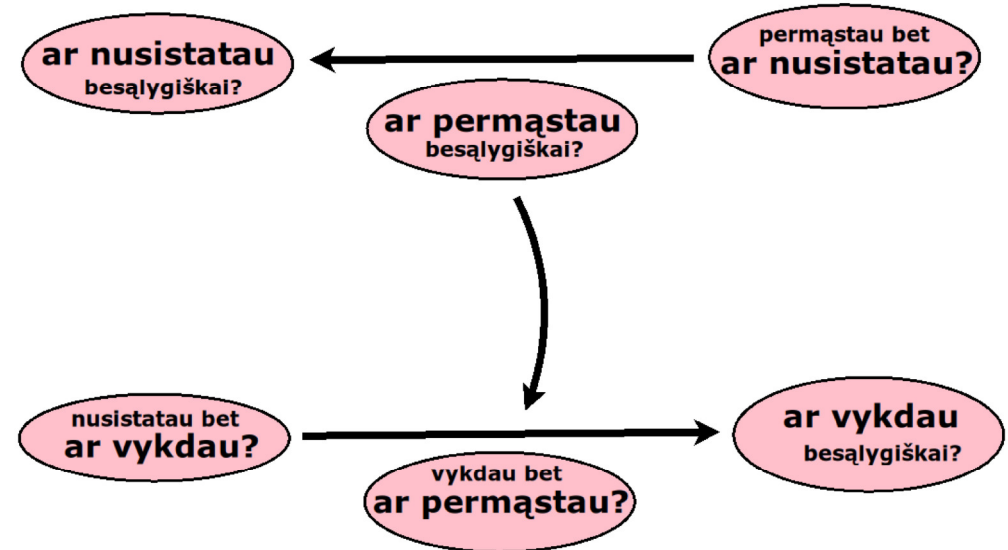
ŽINOJIMAS



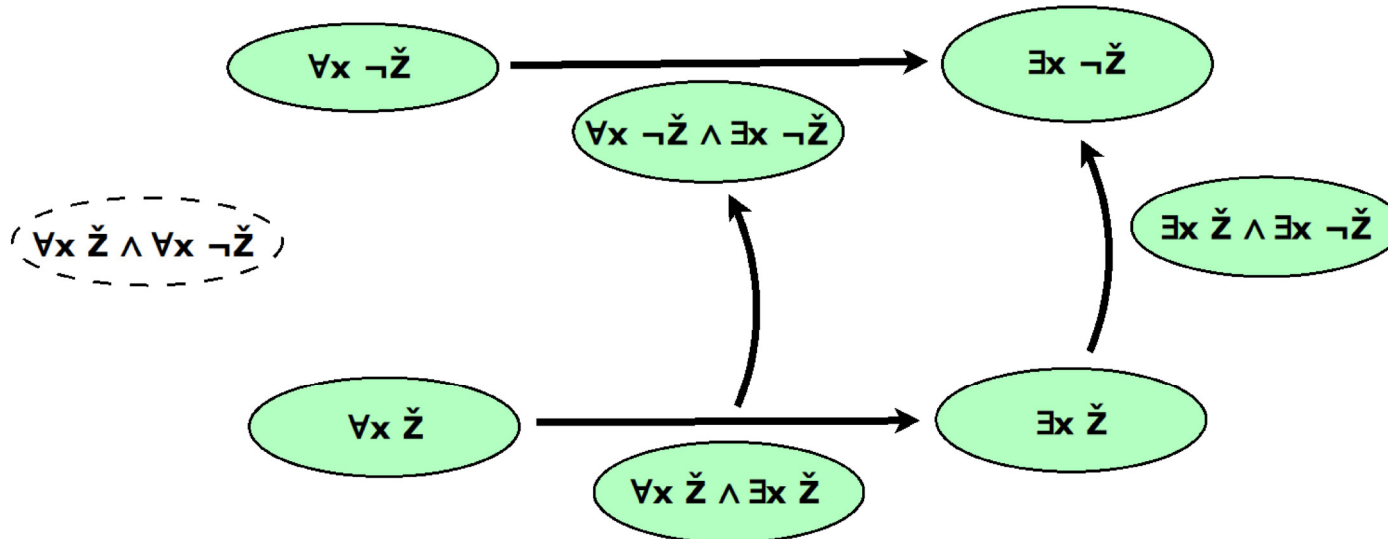
SPRENDIMAS



DOROVĖ



LOGIKA



DIEVAS

*

TVARKA

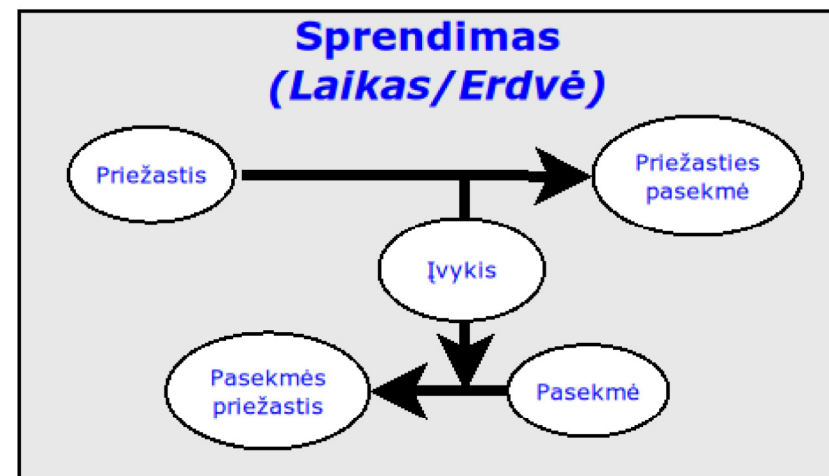
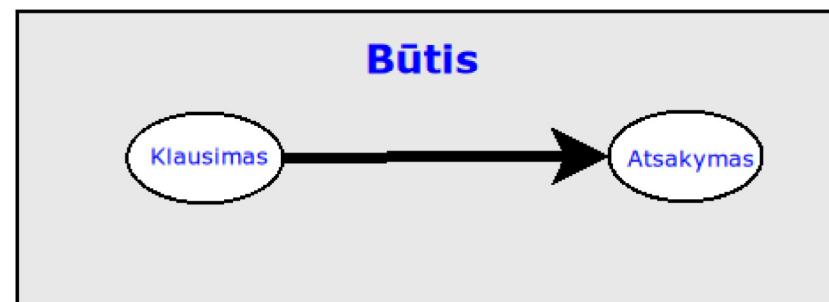
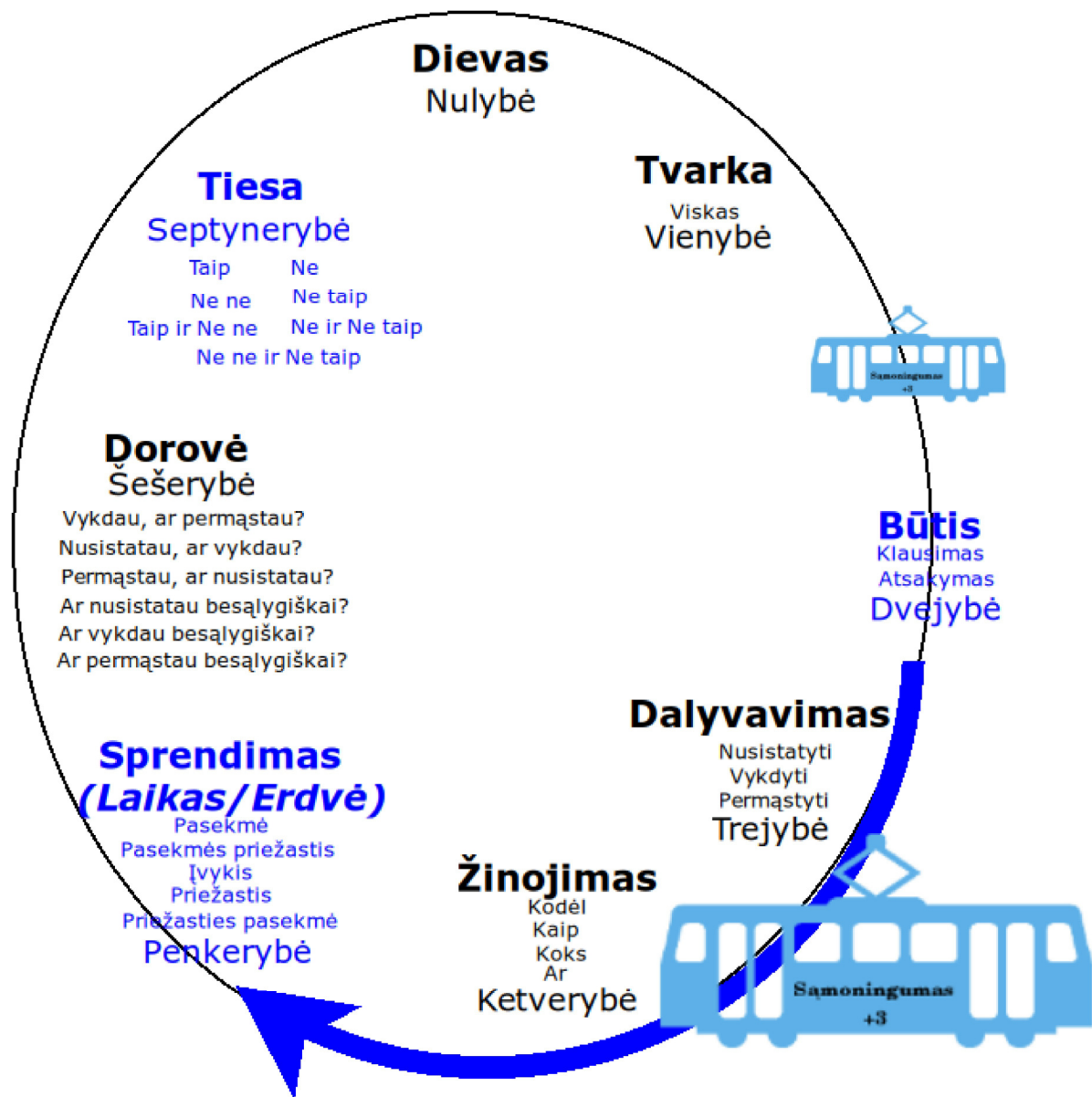
viskas

viskas yra be aplinkos

viskas yra be atrankos

viskas yra be sandaros

viskas yra be pasirinkimo



BOTT PERIODIŠKUMAS

$$\pi_0(O(\infty)) \cong \mathbb{Z}_2$$

$$\pi_1(O(\infty)) \cong \mathbb{Z}_2$$

$$\pi_2(O(\infty)) \cong 0$$

$$\pi_3(O(\infty)) \cong \mathbb{Z}$$

$$\pi_4(O(\infty)) \cong 0$$

$$\pi_5(O(\infty)) \cong 0$$

$$\pi_6(O(\infty)) \cong 0$$

$$\pi_7(O(\infty)) \cong \mathbb{Z}$$

$$\pi_0(U(\infty)) \cong 0$$

$$\pi_1(U(\infty)) \cong \mathbb{Z}$$

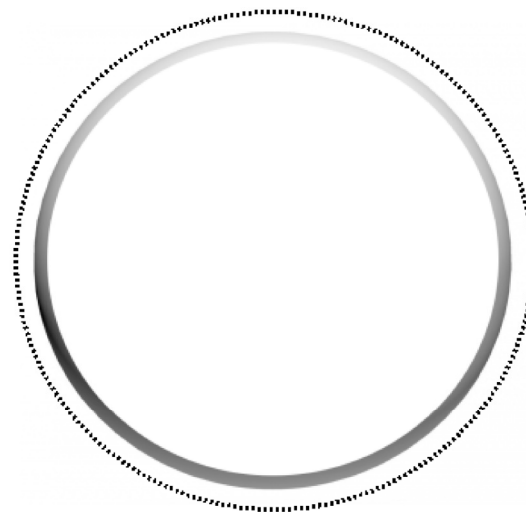
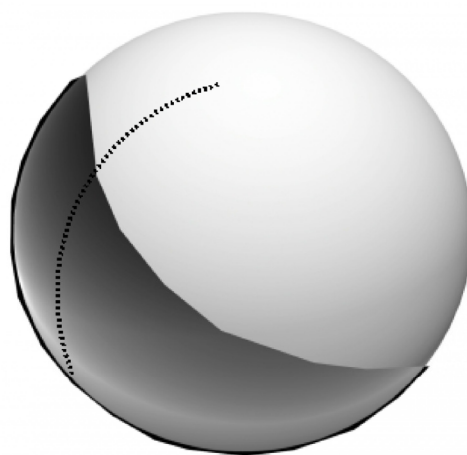
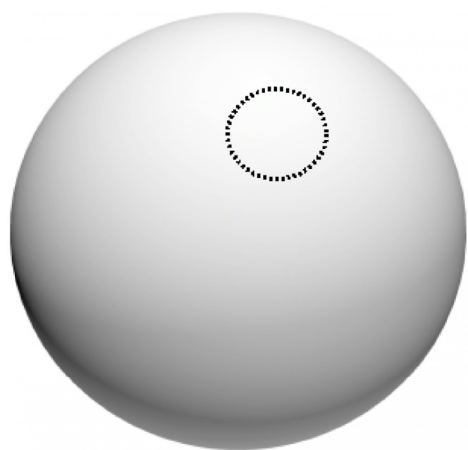
$$\pi_{k+2}(U(\infty)) \cong \pi_k(U(\infty))$$

$$\pi_{k+8}(O(\infty)) \cong \pi_k(O(\infty))$$

$$O(\infty) = \bigcup_{k=1}^{\infty} O(k) \supset \cdots \supset O(n) \supset \cdots \supset O(2) \supset O(1)$$

HOMOTOPIJOS PAVYZDŽIAI

$$\pi_1(S^2) = 0 \quad \pi_1(RP^2) = \mathbb{Z}_2 \quad \pi_1(S^1) = \mathbb{Z}$$



SFERU HOMOTOPIJA

$$S^n \rightarrow S^m$$

$S^n \rightarrow$	S^0	S^1	S^2	S^3	S^4	S^5	S^6	S^7	S^8	S^9	S^{10}	S^{11}	S^{12}	$S^{\geq 13}$
$\pi_{<n}(S^n)$		.	.	.	.	.	.	.	.	.	.	.	.	.
$\pi_{0+n}(S^n)$	2	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
$\pi_{1+n}(S^n)$	.	.	∞	2	2	2	2	2	2	2	2	2	2	2
$\pi_{2+n}(S^n)$	.	.	2	2	2	2	2	2	2	2	2	2	2	2
$\pi_{3+n}(S^n)$	.	.	2	12	$\infty \cdot 12$	24	24	24	24	24	24	24	24	24
$\pi_{4+n}(S^n)$	.	.	12	2	2^2	2	.	.	.	.	.	.	.	.
$\pi_{5+n}(S^n)$	.	.	2	2	2^2	2	∞	.	.	.	.	.	.	.
$\pi_{6+n}(S^n)$	.	.	2	3	$24 \cdot 3$	2	2	2	2	2	2	2	2	2
$\pi_{7+n}(S^n)$	.	.	3	15	15	30	60	120	$\infty \cdot 120$	240	240	240	240	240
$\pi_{8+n}(S^n)$	.	.	15	2	2	2	$24 \cdot 2$	2^3	2^4	2^3	2^2	2^2	2^2	2^2
$\pi_{9+n}(S^n)$	.	.	2	2^2	2^3	2^3	2^3	2^4	2^5	2^4	$\infty \cdot 2^3$	2^3	2^3	2^3
$\pi_{10+n}(S^n)$	.	.	2^2	$12 \cdot 2$	$120 \cdot 12 \cdot 2$	$72 \cdot 2$	$72 \cdot 2$	$24 \cdot 2$	$24^2 \cdot 2$	$24 \cdot 2$	$12 \cdot 2$	$6 \cdot 2$	6	6
$\pi_{11+n}(S^n)$	.	.	$12 \cdot 2$	$84 \cdot 2^2$	$84 \cdot 2^5$	$504 \cdot 2^2$	$504 \cdot 4$	$504 \cdot 2$	$504 \cdot 2$	$504 \cdot 2$	504	504	$\infty \cdot 504$	504

GEOMETRIJAI REIKALINGA VIDINĖ SANDAUGA

$$\mathbf{u} \cdot \mathbf{v} = (Q\mathbf{u}) \cdot (Q\mathbf{v})$$

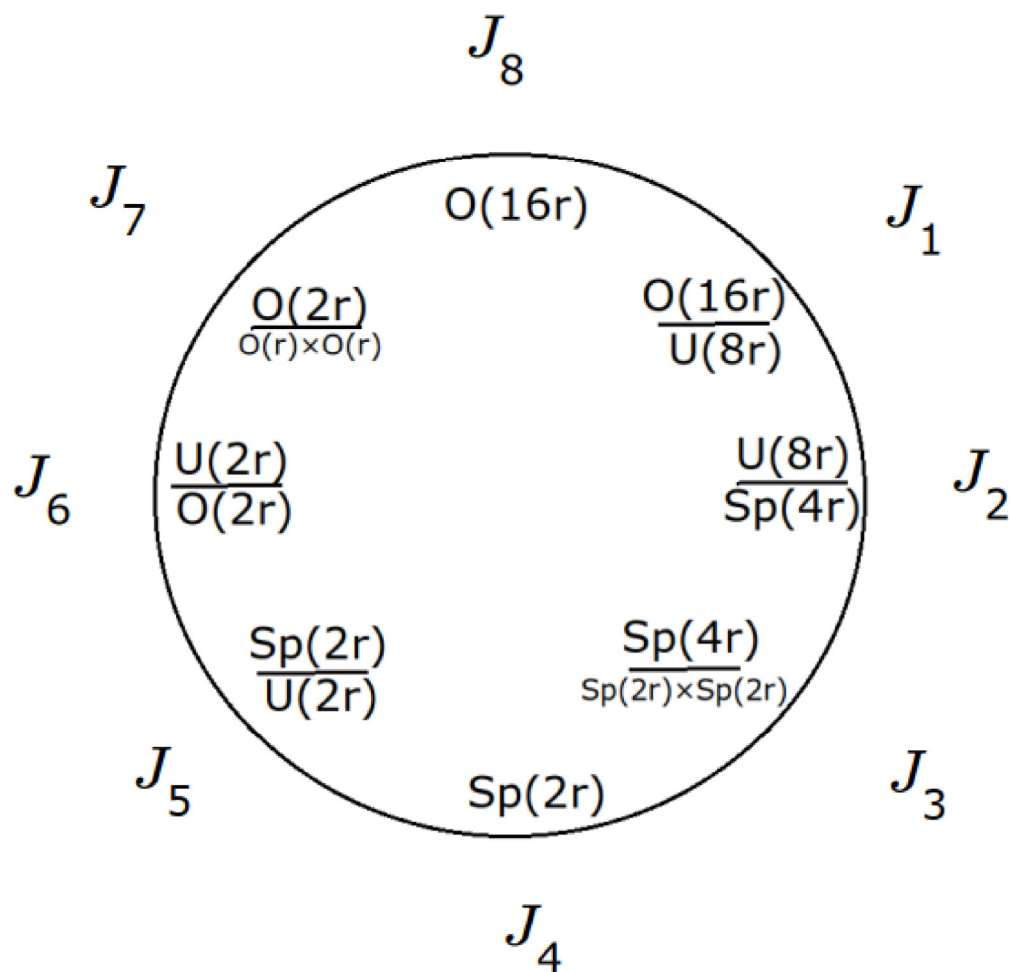
$$Q Q^* T = I \quad \begin{pmatrix} - & - & - \\ - & - & - \\ - & - & - \end{pmatrix} \begin{pmatrix} * & * & * \\ * & * & * \\ * & * & * \end{pmatrix} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \quad Q^{-1} = Q^* T$$

$$Q \in O(n) \quad \text{ORTOGONALI} \quad a^* = a \in \mathbb{R}$$

$$Q \in U(n) \quad \text{UNITARINĖ} \quad (a + bi)^* = a - bi \in \mathbb{C}$$

$$Q \in Sp(n) \quad \text{SIMPLEKTINĖ} \quad (a + bi + cj + dk)^* = a - bi - cj - dk \in \mathbb{H}$$

SIMETRINĖS ERDVĖS



TIESINĖS KOMPLEKSNĖS SANDAROS

požiūriai

$$J_m \in O(16r)$$

$$J_m^2 = -1$$

$$J_m J_n = -J_n J_m$$

$$g \in G_m$$

$$g J_m = J_m g \Rightarrow g \in H_m$$

$$g J_m = -J_m g \Rightarrow g \in \frac{G_m}{H_m}$$

J_1
 $\downarrow$

pašamone $\rightarrow U(k)$

$$\begin{pmatrix} & -1 \\ 1 & \end{pmatrix} \begin{pmatrix} a & -b \\ b & a \end{pmatrix} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \begin{pmatrix} & -1 \\ 1 & \end{pmatrix}$$

$$\begin{pmatrix} & -1 \\ 1 & \end{pmatrix} \begin{pmatrix} a & b \\ b & -a \end{pmatrix} = - \begin{pmatrix} a & b \\ b & -a \end{pmatrix} \begin{pmatrix} & -1 \\ 1 & \end{pmatrix}$$

samone $\rightarrow$ $\begin{pmatrix} a & -b \\ b & a \end{pmatrix} \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \rightarrow \frac{O(2k)}{U(k)}$
posūkis atspindys

HARLANDO PLÈTOTÈ

THUE-MORSE SEKA

$$k_1 = 1$$

$$k_2 = 1, -1$$

$$k_3 = 1, -1, -1, 1$$

$$k_4 = 1, -1, -1, 1, -1, 1, 1, -1$$

$$k_n = k_{n-1} || - k_{n-1}$$

$$K_n = \text{diag}(k_n)$$

$$J_n = \begin{pmatrix} & -K_n \\ K_n & \end{pmatrix}$$

neredukuotas įvaizdis

$$J_1 = \begin{pmatrix} & -1 & & & & \\ 1 & & & & & \\ & & -1 & & & \\ & 1 & & & & \\ & & & -1 & & \\ & & 1 & & & \\ & & & & -1 & \\ & & & & & 1 \end{pmatrix}$$

J_1 generuoja $\mathbb{C}$

$$J_2 = \begin{pmatrix} & & 1 & & & \\ & & & -1 & & \\ -1 & & & & & \\ & 1 & & & & \\ & & & & 1 & \\ & & & -1 & & -1 \\ & & & & 1 & \end{pmatrix}$$

pridėjus J_2 generuoja $\mathbb{H}$

$$J_3 = \begin{pmatrix} & & & -1 & & \\ & & -1 & & & \\ & 1 & & & & \\ 1 & & & & & \\ & & & & 1 & \\ & & & & & 1 \\ & & & -1 & & \\ & & -1 & & & \end{pmatrix}$$

pridėjus J_3 generuoja $\mathbb{H} \oplus \mathbb{H}$

$$J_4 = \begin{pmatrix} & & & & & 1 \\ & & & & 1 & \\ & & & -1 & & \\ & & -1 & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ -1 & & & & & \end{pmatrix}$$

pridėjus J_4 generuoja $M_2(\mathbb{H})$

$$J_5 = \begin{pmatrix} & & & & -1 & \\ & & & & & 1 \\ & & -1 & & & \\ & & & -1 & & \\ & 1 & & & 1 & \\ & & 1 & & & \\ & & & -1 & & \\ 1 & & & & & \\ & -1 & & & & \end{pmatrix}$$

pridėjus J_5 generuoja $M_4(\mathbb{C})$

$$J_6 = \begin{pmatrix} & & & & 1 & \\ & & & 1 & & \\ & & 1 & & & \\ & & & & 1 & \\ & -1 & & & & \\ -1 & & & & & \\ & & -1 & & & \\ & & & -1 & & \end{pmatrix}$$

pridėjus J_6 generuoja $M_8(\mathbb{R})$

$$J_7 = \begin{pmatrix} & & \hat{r} & & & \\ & & & -\hat{r} & & \\ -\hat{r} & & & & & \\ & \hat{r} & & & & \\ & & & & -\hat{r} & \\ & & & \hat{r} & & \\ & & & & & \hat{r} \\ & & -\hat{r} & & & \end{pmatrix}$$

pridėjus J_7 generuoja $M_8(\mathbb{R}) \oplus M_8(\mathbb{R})$

$$J_8 = \begin{pmatrix} & & & & -\hat{r} & \\ & & & & & \hat{r} \\ & & \hat{r} & & & \\ & & & -\hat{r} & & \\ & -\hat{r} & & & & \\ & & \hat{r} & & & \\ \hat{r} & & & & & \\ & -\hat{r} & & & & \end{pmatrix}$$

pridėjus J_8 generuoja $M_{16}(\mathbb{R})$

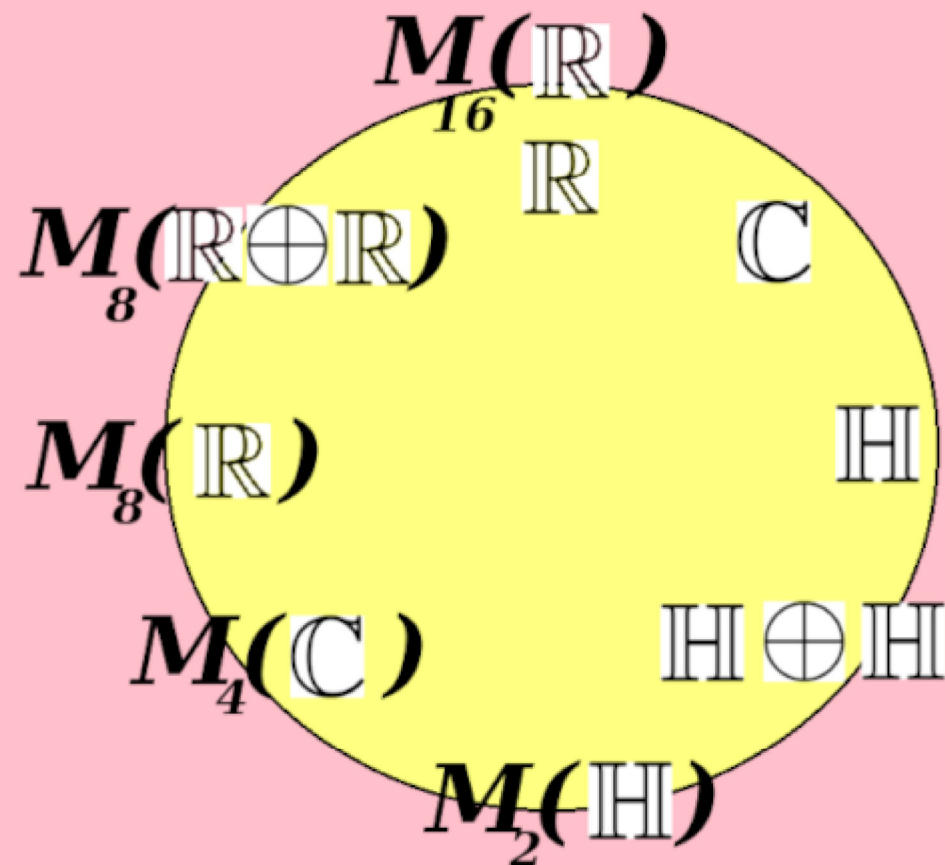
$$\hat{r} = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

Klifordo algebrų generatoriai

požiūriai yra pasirinkimai

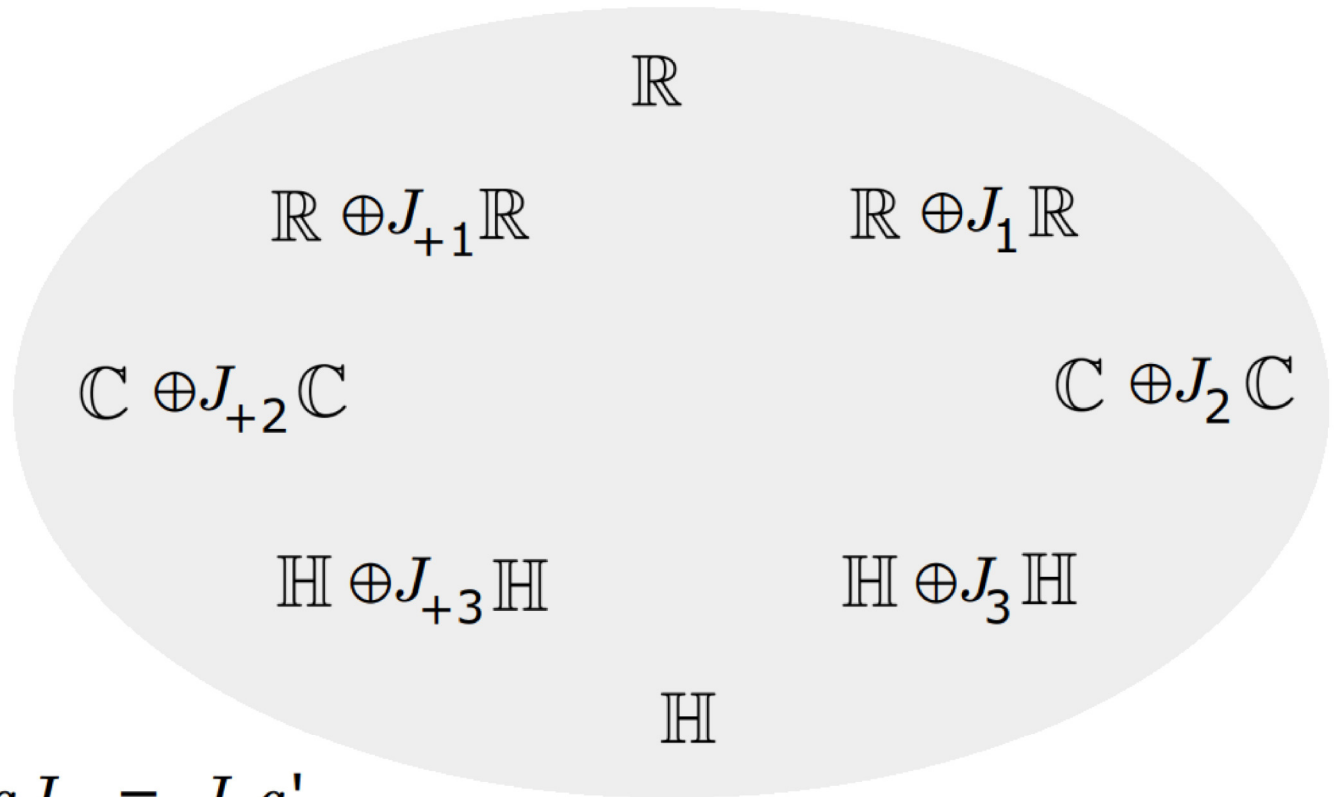
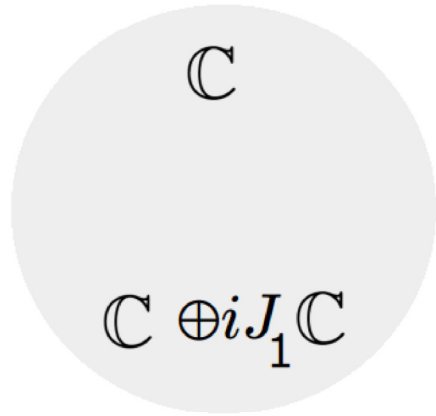
$\mathbb{R}$
$\mathbb{C}$
$\mathbb{H}$
$\mathbb{H} \oplus \mathbb{H}$

				1				
		1				J_1		
	1			J_1	J_2		$J_1 J_2$	
1	J_1	J_2	J_3	$J_1 J_2$	$J_1 J_3$	$J_2 J_3$	$J_1 J_2 J_3$	



SUPER KŪNŲ KLASIFIKACIJA

Hurvico teoremos bendrinys



PASĄMONĖ
poriniai $\mathbb{A}$

SĄMONĖ
neporiniai $J_m \mathbb{A}$

SĄMONINGUMAS
automorfizmas $a \leftrightarrow a' \quad aJ_m = J_m a'$

TOPOLOGIJA IŠREIŠKIA POSLINKIUS

nuošalaus bloko augimas sieja sąmonę ir pasąmonę

$$J_1 = \begin{pmatrix} \boxed{1} & \boxed{-1} & & & & \\ & i & & & & \\ & & 1 & & & \\ & & & -1 & & \\ & & & & 1 & \\ & & & & & -1 \end{pmatrix}$$

J_1 generuoja $\mathbb{C}$

$$J_2 = \begin{pmatrix} & \boxed{1} & & & & \\ & & -1 & & & \\ \boxed{-1} & & & & & \\ & 1 & & j & & \\ & & & & 1 & \\ & & & & & -1 \end{pmatrix}$$

pridėjus J_2 generuoja $\mathbb{H}$

$$J_3 = \begin{pmatrix} & & \boxed{-1} & & & \\ & & & -1 & & \\ \boxed{1} & & & & & \\ & 1 & & & & \\ & & & k & & \\ & & & & 1 & \\ & & & & & -1 \\ & & & & & & -k \end{pmatrix}$$

pridėjus J_3 generuoja $\mathbb{H} \oplus \mathbb{H}$

$$J_4 = \begin{pmatrix} & & & \boxed{\mathbb{Z}} & & \\ & & & & 1 & \\ & & & & & -1 \\ \boxed{-1} & & & & & \\ & 1 & & & & \\ & & -1 & & & \end{pmatrix}$$

pridėjus J_4 generuoja $M_2(\mathbb{H})$

$$J_5 = \begin{pmatrix} & & & \boxed{0} & & \\ & & & & -1 & \\ & & & & & 1 \\ \boxed{1} & & & & & \\ & 1 & & & & \\ & & -1 & & & \end{pmatrix}$$

pridėjus J_5 generuoja $M_4(\mathbb{C})$

$$J_6 = \begin{pmatrix} & & & & \boxed{1} & \\ & & & & & 0 \\ & & & & & & 1 \\ \boxed{-1} & & & & & \\ & -1 & & & & \\ & & -1 & & & \end{pmatrix}$$

pridėjus J_6 generuoja $M_8(\mathbb{R})$

$$J_7 = \begin{pmatrix} & & \boxed{\hat{r}} & & & \\ & & & -\hat{r} & & \\ \boxed{-\hat{r}} & & & & & \\ & \hat{r} & & & & \\ & & & -\hat{r} & & \\ & & & & \hat{r} & \\ & & & & & -\hat{r} \end{pmatrix}$$

pridėjus J_7 generuoja $M_8(\mathbb{R}) \oplus M_8(\mathbb{R})$

$$J_8 = \begin{pmatrix} & & & \boxed{\mathbb{Z}} & & \\ & & & & -\hat{r} & \\ & & & & & \hat{r} \\ \boxed{\hat{r}} & & & & & \\ & -\hat{r} & & & & \\ & & \hat{r} & & & \\ & & & -\hat{r} & & \end{pmatrix}$$

pridėjus J_8 generuoja $M_{16}(\mathbb{R})$

$$\hat{r} = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

$$J_1^2 = -1$$

V SKYLA

$$(J_1 J_2)^2 = J_1 J_2 J_1 J_2 = -J_1^2 J_2^2 = -1$$

$$(J_1 J_2 J_3)^2 = J_1 J_2 J_3 J_1 J_2 J_3 = -J_1 J_2 J_1 J_2 = 1$$

$$(J_1 J_2 J_3)^2 - 1 = 0 \quad \Rightarrow \quad V = V_{+1} + V_{-1}$$

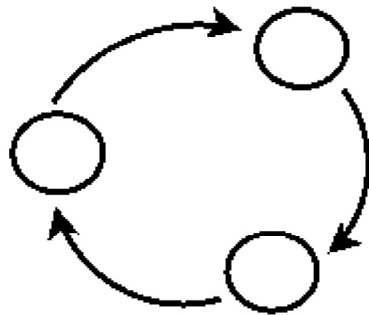
$$(J_1 J_2 J_3) V_{+1} = V_{+1} \quad \text{ir} \quad (J_1 J_2 J_3) V_{-1} = -V_{-1}$$

$$J_1 J_2 J_3 = 1$$

$$J_1 J_2 = -J_3$$

$$J_2 J_1 = J_3$$

$$j \ i = k$$



$$J_1 J_2 J_3 = -1$$

$$J_1 J_2 = J_3$$

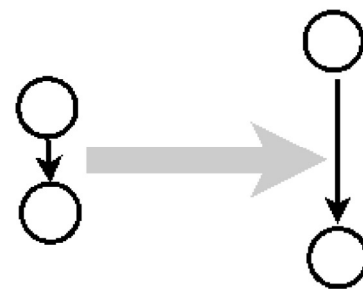
$$i \ j = k$$

ISOMETRIJA L

$$J_1 J_2 J_3 = \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -1 & \\ & & & & & -1 \\ & & & & & & -1 \\ & & & & & & & -1 \end{pmatrix} \quad V = \begin{pmatrix} V_{+1} \\ V_{-1} \end{pmatrix} \quad L = \begin{pmatrix} & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \\ & & & & & & 1 \\ -1 & & & & & & \\ & -1 & & & & & \\ & & -1 & & & & \\ & & & -1 & & & \end{pmatrix}$$

$$J_1 J_2 J_3 V_{+1} = V_{+1}$$

$$J_1 J_2 J_3 V_{-1} = -V_{-1}$$



L permeta J_1 ir J_2

$$L V_{+1} = -V_{-1}$$

$$L V_{-1} = V_{+1}$$

OKTONIONAI $\mathbb{O}$

generatoriai $1 \ i \ j \ k \ L \ Li \ Lj \ Lk$

neasociatyvūs $(La)b = L(ba)$ **žinojimo veidrodis** L

$\mathbb{O}\mathbb{P}^1$ **kanoninis tiesių kompleksas** $[l_{\mathbb{O}}]$ **generuoja Bott periodiškumą**

$$J_1$$

$$J_2$$

$$J_3 = J_2 J_1 \operatorname{diag}[1 \ 1 \ 1 \ 1 \ -1 \ -1 \ -1 \ -1]$$

$$J_4 = LJ_2 J_1 \operatorname{diag}[1 \ 1 \ 1 \ 1 \ -1 \ -1 \ -1 \ -1]$$

$$J_5 = LJ_2 \operatorname{diag}[1 \ 1 \ -1 \ -1 \ 1 \ 1 \ -1 \ -1]$$

$$J_6 = LJ_1 \operatorname{diag}[1 \ -1 \ 1 \ -1 \ 1 \ -1 \ 1 \ -1]$$

$$J_7 = L \operatorname{diag}[1 \ -1 \ -1 \ 1 \ 1 \ -1 \ -1 \ 1 \ -1 \ 1 \ 1 \ -1 \ -1 \ 1 \ 1 \ -1]$$

$$\begin{array}{ccc} J_9 & \text{VERČIA PERDARYTI} & J_1, J_2, \dots, J_8 \\ 32 \times 32 & & 16 \times 16 \end{array}$$

$$\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} J_i & \\ & J_i \end{pmatrix} = - \begin{pmatrix} J_i & \\ & J_i \end{pmatrix} \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

$$m_{ab}J_i = -J_i m_{ab}$$

$$\textit{antikomutuoja su visomis matricomis} \quad J_1, J_2, \dots, J_8$$

o tai neįmanoma!

$$\mathbf{J}_9 = \begin{pmatrix} & -I_{16 \times 16} \\ I_{16 \times 16} & \end{pmatrix}$$

$$\begin{pmatrix} & -I \\ I & \end{pmatrix} \begin{pmatrix} J_i & \\ & -J_i \end{pmatrix} = - \begin{pmatrix} J_i & \\ & -J_i \end{pmatrix} \begin{pmatrix} & -I \\ I & \end{pmatrix}$$

*tenka perdaryti visas įstrižaines $\mathbf{J}_i$, kad jos save paneigtų $-\mathbf{J}_i$
ir teks vis naujai jas perdaryti!*

LIE GRUPĖS

$$G_k \supset H_k$$

$$\Psi \qquad \Psi$$

$$e^{gt} \qquad e^{ht}$$

SIMETRINĖ
ERDVĖ

$$\frac{G_k}{H_k}$$

$$\Psi$$

$$e^{iHt}$$

laikas $t \in \mathbb{R}$

LIE ALGEBROS

$$\mathfrak{g}_k = m_k + h_k$$

$$\Psi \qquad \Psi \qquad \Psi$$

$$g = iH + h$$



HAMILTONIANAS

$$(e^q)^T = (e^q)^{-1}$$

$O(r)$ ORTOGONALI $\mathfrak{o}(r)$

$$q^T = -q$$

antisimetrinė

$$((e^q)^*)^T = (e^q)^{-1}$$

$U(r)$ UNITARINĖ $\mathfrak{u}(r)$

$$q^T = -q^*$$

antiermitinė

$$((e^q)^{\otimes})^T = (e^q)^{-1}$$

$Sp(r)$ SIMPLEKTINĖ $\mathfrak{sp}(r)$

$$q^T = -q^{\otimes}$$

kvaternioninė antiermitinė

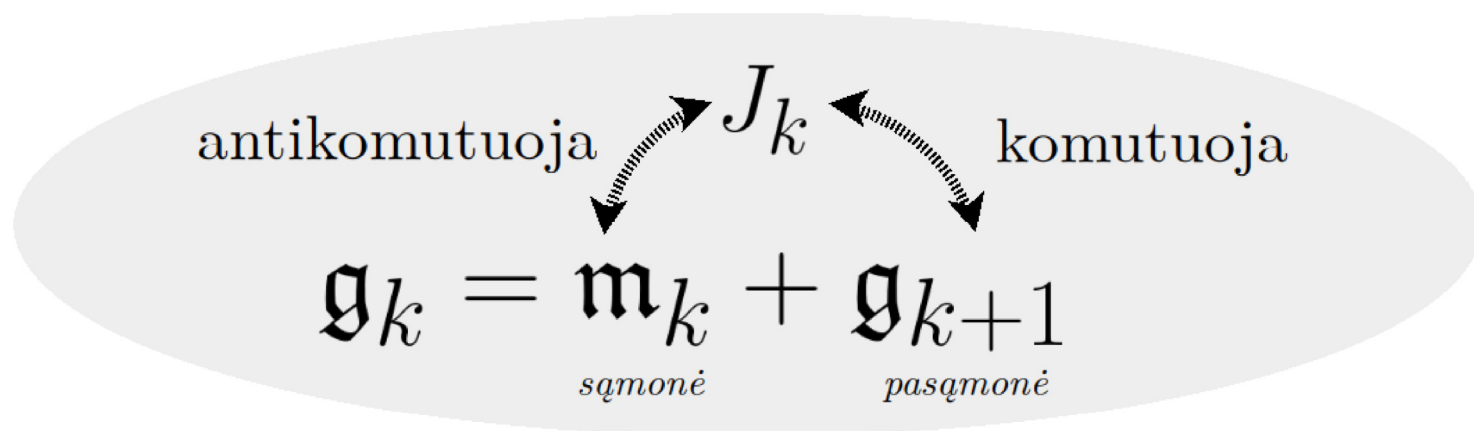
VISKO PADALINIMAI

antisimetrinės matricos

$$\mathfrak{o}(16r) \supset \mathfrak{u}(8r) \supset \mathfrak{sp}(4r) \supset \mathfrak{sp}(2r) \times \mathfrak{sp}(2r) \supset \mathfrak{sp}(2r) \supset \mathfrak{u}(2r) \supset \mathfrak{o}(2r) \supset \mathfrak{o}(r) \times \mathfrak{o}(r) \supset \mathfrak{o}(r)$$

$$\mathfrak{o}(16r) = \mathfrak{g}_0$$

viskas



$$\mathfrak{g}_0 - \mathfrak{g}_k = \underbrace{\mathfrak{m}_0}_{i\mathcal{H}_0} + \underbrace{\mathfrak{m}_1}_{i\mathcal{H}_1} + \cdots + \underbrace{\mathfrak{m}_{k-1}}_{i\mathcal{H}_{k-1}}$$

visko padalinimas į k požiūrius

Lie algebrų skaidymas skaidiklis	$\mathfrak{o}(2M) = \mathfrak{u}(M) + \mathfrak{m}$ J_1	$\mathfrak{u}(2M) = \mathfrak{sp}(M) + \mathfrak{m}$ J_2
poalgebrių 2×2 blokai	$\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ $a + bi \in \mathbb{C}$	$\begin{pmatrix} X & Y \\ -\overline{Y} & \overline{X} \end{pmatrix}$ $X + Yj \in \mathbb{H}$
liekanos 2×2 blokai	$\begin{pmatrix} a & b \\ b & -a \end{pmatrix}$ $a, b \in \mathbb{R}$	$\begin{pmatrix} X & Y \\ \overline{Y} & -\overline{X} \end{pmatrix}$ $X, Y \in \mathbb{C}$
Hamiltoniano pavidalas	$\begin{pmatrix} & \mathbf{h}_{MM} \\ -\mathbf{h}_{MM}^* & \end{pmatrix}$ $m = \mathbf{h}_{MM} = -\mathbf{h}_{MM}^*$	$\begin{pmatrix} \mathbf{h}_{aa} & \mathbf{h}_{ab} \\ -\mathbf{h}_{ab}^* & \mathbf{h}_{aa}^* \end{pmatrix}$
kvantinės simetrijos	$C^2 = +1, T^2 = -1, S^2 = 1$	$T^2 = -1$

Lie algebrų skaidymas skaidiklis	$\mathfrak{sp}(2M) = \mathfrak{sp}(M) \oplus \mathfrak{sp}(M) + \mathfrak{m}$ J_3	$\mathfrak{sp}(M) \oplus \mathfrak{sp}(M) = \mathfrak{sp}(M) + \mathfrak{m}$ J_4
poalgebrių 2×2 blokai	$\begin{pmatrix} q_1 & \\ & q_2 \end{pmatrix}$ $q_1, q_2 \in \mathbb{H}$	$\begin{pmatrix} q & \\ & q \end{pmatrix}$ $q \in \mathbb{H}$
liekanos 2×2 blokai	$\begin{pmatrix} & q_1 \\ q_2 & \end{pmatrix}$ $q_1, q_2 \in \mathbb{H}$	$\begin{pmatrix} q & \\ & -q \end{pmatrix}$ $q \in \mathbb{H}$
Hamiltoniano pavidalas	$\begin{pmatrix} \mathbf{h}_{rr} & \mathbf{h}_{rs} \\ -\mathbf{h}_{rs}^* & \mathbf{h}_{rr}^* \\ \text{h.c} & \end{pmatrix}$	$\begin{pmatrix} \mathbf{h}_{aa} & \mathbf{h}_{ab} \\ \mathbf{h}_{ab}^* & -\mathbf{h}_{aa}^* \end{pmatrix}$
kvantinės simetrijos	$T^2 = -1, C^2 = -1, S^2 = 1$	$C^2 = -1$

Lie algebrų skaidymas
skaidiklis

$$\mathfrak{sp}(M) = \mathfrak{u}(M) + \mathfrak{m}$$

$$J_5$$

$$\mathfrak{u}(M) = \mathfrak{o}(M) + \mathfrak{m}$$

$$J_6$$

poalgebrių 2×2 blokai

$$\begin{pmatrix} z & \\ & z \end{pmatrix}$$

$$z \in \mathbb{C} \subset \mathbb{H}$$

$$\begin{pmatrix} a & \\ & a \end{pmatrix}$$

$$a \in \mathbb{R} \subset \mathbb{H}$$

liekanos 2×2 blokai

$$\begin{pmatrix} zj & \\ & zj \end{pmatrix}$$

$$z = a + bi \in \mathbb{C} \subset \mathbb{H}$$

$$j \in \mathbb{H}$$

$$\begin{pmatrix} bi & \\ & bi \end{pmatrix}$$

$$b \in \mathbb{R} \subset \mathbb{H}, i \in \mathbb{H}$$

Hamiltoniano pavidalas

$$\begin{pmatrix} & \mathbf{h}_{MM} \\ \mathbf{h}_{MM}^* & \end{pmatrix}$$

$$\mathbf{h}_{MM} = \mathbf{h}_{MM}^T$$

$$\mathbf{H} = \mathbf{H}^*$$

kvantinės simetrijos

$$C^2 = -1, T^2 = +1, S^2 = 1$$

$$T^2 = +1$$

Lie algebrų skaidymas skaidiklis	$\mathfrak{o}(2M) = \mathfrak{o}(M) \oplus \mathfrak{o}(M) + \mathfrak{m}$	$\mathfrak{o}(M) \oplus \mathfrak{o}(M) = \mathfrak{o}(M) + \mathfrak{m}$
	J_7	J_8
poalgebrių 2×2 blokai	$\begin{pmatrix} a_1 & \\ & a_2 \end{pmatrix}$	$\begin{pmatrix} a & \\ & a \end{pmatrix}$
	$a_1, a_2 \in \mathbb{R} \subset M_8(\mathbb{R})$	$a \in \mathbb{R} \subset M_8(\mathbb{R})$
liekanos 2×2 blokai	$\begin{pmatrix} & a_1 \\ a_2 & \end{pmatrix}$	$\begin{pmatrix} a & \\ & -a \end{pmatrix}$
	$a_1, a_2 \in \mathbb{R} \subset M_8(\mathbb{R})$	$a \in \mathbb{R} \subset M_8(\mathbb{R})$
Hamiltoniano pavidalas	$\begin{pmatrix} & \mathbf{h}_{pq} \\ \mathbf{h}_{pq}^T & \end{pmatrix}$	$\mathbf{H} = -\mathbf{H}^* = \begin{pmatrix} & -m \\ m & \end{pmatrix}$
	$\mathbf{h}_{pq} = \mathbf{h}_{pq}^*$	
kvantinės simetrijos	$C^2 = +1, T^2 = +1, S^2 = 1$	$C^2 = +1$

$$\mathfrak{o}(2M) = \mathfrak{u}(M) + \mathfrak{m}$$

$$J_1$$

$$\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$$

$$a + bi \in \mathbb{C}$$

$$\begin{pmatrix} a & b \\ b & -a \end{pmatrix}$$

$$a, b \in \mathbb{R}$$

$$\begin{pmatrix} & \mathbf{h}_{MM} \\ -\mathbf{h}_{MM}^* & \end{pmatrix}$$

$$m = \mathbf{h}_{MM} = -\mathbf{h}_{MM}^*$$

$$C^2 = +1, T^2 = -1, S^2 = 1$$

$$\mathfrak{u}(2M) = \mathfrak{sp}(M) + \mathfrak{m}$$

$$J_2$$

$$\begin{pmatrix} X & Y \\ -\bar{Y} & \bar{X} \end{pmatrix}$$

$$X + Yj \in \mathbb{H}$$

$$\begin{pmatrix} X & Y \\ \bar{Y} & -\bar{X} \end{pmatrix}$$

$$X, Y \in \mathbb{C}$$

$$\begin{pmatrix} \mathbf{h}_{aa} & \mathbf{h}_{ab} \\ -\mathbf{h}_{ab}^* & \mathbf{h}_{aa}^* \end{pmatrix}$$

$$T^2 = -1$$

$$\mathfrak{sp}(2M) = \mathfrak{sp}(M) \oplus \mathfrak{sp}(M) + \mathfrak{m}$$

$$J_3$$

$$\begin{pmatrix} q_1 & \\ & q_2 \end{pmatrix}$$

$$q_1, q_2 \in \mathbb{H}$$

$$\begin{pmatrix} & q_1 \\ q_2 & \end{pmatrix}$$

$$q_1, q_2 \in \mathbb{H}$$

$$\begin{pmatrix} & \mathbf{h}_{rr} & \mathbf{h}_{rs} \\ & -\mathbf{h}_{rs}^* & \mathbf{h}_{rr}^* \\ \text{h.c} & & \end{pmatrix}$$

$$T^2 = -1, C^2 = -1, S^2 = 1$$

$$\mathfrak{sp}(M) \oplus \mathfrak{sp}(M) = \mathfrak{sp}(M) + \mathfrak{m}$$

$$J_4$$

$$\begin{pmatrix} q & \\ & q \end{pmatrix}$$

$$q \in \mathbb{H}$$

$$\begin{pmatrix} q & \\ & -q \end{pmatrix}$$

$$q \in \mathbb{H}$$

$$\begin{pmatrix} \mathbf{h}_{aa} & \mathbf{h}_{ab} \\ \mathbf{h}_{ab}^* & -\mathbf{h}_{aa}^* \end{pmatrix}$$

$$C^2 = -1$$

$$\mathfrak{sp}(M) = \mathfrak{u}(M) + \mathfrak{m}$$

$$J_5$$

$$\begin{pmatrix} z & \\ & z \end{pmatrix}$$

$$z \in \mathbb{C} \subset \mathbb{H}$$

$$\begin{pmatrix} zj & \\ & zj \end{pmatrix}$$

$$z = a + bi \in \mathbb{C} \subset \mathbb{H}$$

$$j \in \mathbb{H}$$

$$\begin{pmatrix} & \mathbf{h}_{MM} \\ \mathbf{h}_{MM}^* & \end{pmatrix}$$

$$\mathbf{h}_{MM} = \mathbf{h}_{MM}^T$$

$$C^2 = -1, T^2 = +1, S^2 = 1$$

$$\mathfrak{u}(M) = \mathfrak{o}(M) + \mathfrak{m}$$

$$J_6$$

$$\begin{pmatrix} a & \\ & a \end{pmatrix}$$

$$a \in \mathbb{R} \subset \mathbb{H}$$

$$\begin{pmatrix} bi & \\ & bi \end{pmatrix}$$

$$b \in \mathbb{R} \subset \mathbb{H}, i \in \mathbb{H}$$

$$\mathbf{H} = \mathbf{H}^*$$

$$T^2 = +1$$

$$\mathfrak{o}(2M) = \mathfrak{o}(M) \oplus \mathfrak{o}(M) + \mathfrak{m}$$

$$J_7$$

$$\begin{pmatrix} a_1 & \\ & a_2 \end{pmatrix}$$

$$a_1, a_2 \in \mathbb{R} \subset M_8(\mathbb{R})$$

$$\begin{pmatrix} & a_1 \\ a_2 & \end{pmatrix}$$

$$a_1, a_2 \in \mathbb{R} \subset M_8(\mathbb{R})$$

$$\begin{pmatrix} & \mathbf{h}_{pq} \\ \mathbf{h}_{pq}^T & \end{pmatrix}$$

$$\mathbf{h}_{pq} = \mathbf{h}_{pq}^*$$

$$C^2 = +1, T^2 = +1, S^2 = 1$$

$$\mathfrak{o}(M) \oplus \mathfrak{o}(M) = \mathfrak{o}(M) + \mathfrak{m}$$

$$J_8$$

$$\begin{pmatrix} a & \\ & a \end{pmatrix}$$

$$a \in \mathbb{R} \subset M_8(\mathbb{R})$$

$$\begin{pmatrix} a & \\ & -a \end{pmatrix}$$

$$a \in \mathbb{R} \subset M_8(\mathbb{R})$$

$$\mathbf{H} = -\mathbf{H}^* = \begin{pmatrix} & -m \\ m & \end{pmatrix}$$

$$C^2 = +1$$

HAMILTONIANAS H ŽYMI TAPATYBĘ

rišlus

vystosi laike

aprėpia savo raiškos galimybes

veikia savo aplinką, yra jos veikiamas

$\langle Hx, y \rangle = \langle x, Hy \rangle$ *savijungis*

$H^T = H^*$ *Ermito matrica*

HAMILTONIANAS H

$$\mathcal{H} = \mathbf{H}^{(0)} + \Psi^{*T} \mathbf{H}^{(1)} \Psi$$

$$\mathcal{H} = [\psi_1^{*T} \quad \psi_2^{*T}] \begin{pmatrix} E_1 & \\ & E_2 \end{pmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}$$

$$H \Leftrightarrow -H$$

*pervadina
daleles ir skyles*

$$H \Leftrightarrow H^T = H^*$$

*išverčia priežastingumą
(įsijaučiame - atsitokėjame)*

$\psi_2^\dagger$ 

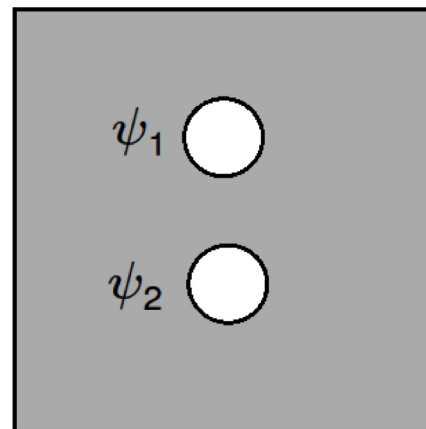
$\psi_1^\dagger$ 

dalelė $E > 0$
atsakymas

$$|i\rangle \equiv \psi_i^{*T} |0\rangle$$

.....

uždrausta $E = 0$



skylė $E < 0$
klausimas

SIMETRIJOS

tiesinė

antitiesinė

įprastoji

laiko apsukimas T

$$U_T H U_T^{*T} = H^*$$

$$(U_T K) H (U_T K)^{*T} = H$$

transponuojanti

krūvio sukeitimas C

$$U_C H U_C^{*T} = -H^*$$

$$(U_C K) H (U_C K)^{*T} = -H$$

veidroдинis atspindys S

$$U_S H U_S^{*T} = -H$$

$$|\psi\rangle = \sum_j c_j |\psi_j\rangle, |\psi'\rangle = \sum_j c'_j |\psi_j\rangle$$

VIGNERIO TEOREMA

$$|c_j|^2 = |c'_j|^2 \text{ nes } |\langle \psi_j | \psi \rangle|^2 = |\langle \psi_j | \psi' \rangle|^2$$

Hilberto erdvės H, K

$$|c_j + c_1|^2 = |c'_j + c'_1|^2$$

Hilberto spindulių erdvės $P(H), P(K)$

simetrinė transformacija $T: P(H) \rightarrow P(K)$

$$|1 + \frac{c_1}{c_j}|^2 = |1 + \frac{c'_1}{c'_j}|^2$$

$$c'_j = c_j e^{i\theta_j}, c'_1 = c_1 e^{i\theta_1}, \frac{c'_1}{c'_j} = \frac{c_1}{c_j} e^{i(\theta_1 - \theta_j)} \text{ ir tegul } \beta = \theta_1 - \theta_j$$

$$\frac{c_1}{c_j} = e^{i\alpha}$$

Egzistuoja simetrinė transformacija $V: H \rightarrow K$

$V: H \rightarrow K$ tokia, kad $T: \Psi \rightarrow V\Psi$

$$|1 + e^{i\alpha}|^2 = |1 + e^{i(\alpha + \beta)}|^2$$

$$(1 + \cos \alpha)^2 + \sin^2 \alpha = (1 + \cos(\alpha + \beta))^2 + \sin^2(\alpha + \beta)$$

$$1 + 2 \cos \alpha = 1 + 2 \cos(\alpha + \beta)$$

ir V unitarinė $\langle V\Psi, V\Phi \rangle = \langle \Psi, \Phi \rangle$
arba antiunitarinė $\langle V\Psi, V\Phi \rangle = \langle \Psi, \Phi \rangle^*$

$$\cos \alpha = \cos(\alpha + \beta) \text{ vadinasi } \operatorname{Re} \frac{c_1}{c_j} = \operatorname{Re} \frac{c'_1}{c'_j} \text{ tad } \beta = 0 + 2\pi \text{ unitarinė arba } \beta = -2\alpha + 2\pi \text{ antiunitarinė}$$

NATURALI BAZĖ PRIKLAUSO NUO SIMETRIJŲ

$$\mathbf{1}_{p,q} = \text{diag}[1, \dots, 1, -1, \dots, -1]$$

$p \qquad q$

$$\mathbf{J}_m = \begin{pmatrix} & \mathbf{1}_m \\ -\mathbf{1}_m & \end{pmatrix}$$

santykiniam apibrėžimui

$$T^2 = \mathbf{1}$$

$$T^2 = -\mathbf{1}$$

$$C^2 = \mathbf{1}$$

$$C^2 = -\mathbf{1}$$

$$U_T = \mathbf{1}_{p,q}$$

$$U_T = \mathbf{J}_m$$

$$U_C = \mathbf{1}_{p,q}$$

$$U_C = \mathbf{J}_m$$

besąlygiškam apibrėžimui

$$U_S = \mathbf{1}_{p,q}$$

$$T^2 = \mathbf{1}$$

$$T^2 = \mathbf{1}$$

$$T^2 = -\mathbf{1}$$

$$T^2 = -\mathbf{1}$$

$$C^2 = \mathbf{1}$$

$$C^2 = -\mathbf{1}$$

$$C^2 = \mathbf{1}$$

$$C^2 = -\mathbf{1}$$

$$U_T = \mathbf{1}_{p,q}$$

$$U_T = \begin{pmatrix} & \mathbf{1}_{MM} \\ \mathbf{1}_{MM} & \end{pmatrix}$$

$$U_T = \mathbf{J}_m$$

$$U_T = \begin{pmatrix} \mathbf{J}_p & \\ & \mathbf{J}_q \end{pmatrix}$$

$$U_C = \mathbf{1}$$

$$U_C = -\mathbf{J}_m$$

$$U_C = \begin{pmatrix} & \mathbf{1}_{MM} \\ \mathbf{1}_{MM} & \end{pmatrix}$$

$$U_C = \begin{pmatrix} -\mathbf{J}_p & \\ & \mathbf{J}_q \end{pmatrix}$$

$$\text{SHS}^{\ominus} - \text{H}$$

$$\mathfrak{o}(2M) = \mathfrak{u}(M) + \mathfrak{m}$$

$$J_1$$

$$\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$$

$$a + bi \in \mathbb{C}$$

$$\begin{pmatrix} a & b \\ b & -a \end{pmatrix}$$

$$a, b \in \mathbb{R}$$

$$\begin{pmatrix} & \mathbf{h}_{MM} \\ -\mathbf{h}_{MM}^* & \end{pmatrix}$$

$$m = \mathbf{h}_{MM} = -\mathbf{h}_{MM}^*$$

$$C^2 = +1, T^2 = -1, S^2 = 1$$

$$\mathfrak{u}(2M) = \mathfrak{sp}(M) + \mathfrak{m}$$

$$J_2$$

$$\begin{pmatrix} X & Y \\ -\bar{Y} & \bar{X} \end{pmatrix}$$

$$X + Yj \in \mathbb{H}$$

$$\begin{pmatrix} X & Y \\ \bar{Y} & -\bar{X} \end{pmatrix}$$

$$X, Y \in \mathbb{C}$$

$$\begin{pmatrix} \mathbf{h}_{aa} & \mathbf{h}_{ab} \\ -\mathbf{h}_{ab}^* & \mathbf{h}_{aa}^* \end{pmatrix}$$

$$T^2 = -1$$

$$\mathfrak{sp}(2M) = \mathfrak{sp}(M) \oplus \mathfrak{sp}(M) + \mathfrak{m}$$

$$J_3$$

$$\begin{pmatrix} q_1 & \\ & q_2 \end{pmatrix}$$

$$q_1, q_2 \in \mathbb{H}$$

$$\begin{pmatrix} & q_1 \\ q_2 & \end{pmatrix}$$

$$q_1, q_2 \in \mathbb{H}$$

$$\begin{pmatrix} & \mathbf{h}_{rr} & \mathbf{h}_{rs} \\ & -\mathbf{h}_{rs}^* & \mathbf{h}_{rr}^* \\ \text{h.c} & & \end{pmatrix}$$

$$T^2 = -1, C^2 = -1, S^2 = 1$$

$$\mathfrak{sp}(M) \oplus \mathfrak{sp}(M) = \mathfrak{sp}(M) + \mathfrak{m}$$

$$J_4$$

$$\begin{pmatrix} q & \\ & q \end{pmatrix}$$

$$q \in \mathbb{H}$$

$$\begin{pmatrix} q & \\ & -q \end{pmatrix}$$

$$q \in \mathbb{H}$$

$$\begin{pmatrix} \mathbf{h}_{aa} & \mathbf{h}_{ab} \\ \mathbf{h}_{ab}^* & -\mathbf{h}_{aa}^* \end{pmatrix}$$

$$C^2 = -1$$

$$\mathfrak{sp}(M) = \mathfrak{u}(M) + \mathfrak{m}$$

$$J_5$$

$$\begin{pmatrix} z & \\ & z \end{pmatrix}$$

$$z \in \mathbb{C} \subset \mathbb{H}$$

$$\begin{pmatrix} zj & \\ & zj \end{pmatrix}$$

$$z = a + bi \in \mathbb{C} \subset \mathbb{H}$$

$$j \in \mathbb{H}$$

$$\begin{pmatrix} & \mathbf{h}_{MM} \\ \mathbf{h}_{MM}^* & \end{pmatrix}$$

$$\mathbf{h}_{MM} = \mathbf{h}_{MM}^T$$

$$C^2 = -1, T^2 = +1, S^2 = 1$$

$$\mathfrak{u}(M) = \mathfrak{o}(M) + \mathfrak{m}$$

$$J_6$$

$$\begin{pmatrix} a & \\ & a \end{pmatrix}$$

$$a \in \mathbb{R} \subset \mathbb{H}$$

$$\begin{pmatrix} bi & \\ & bi \end{pmatrix}$$

$$b \in \mathbb{R} \subset \mathbb{H}, i \in \mathbb{H}$$

$$\mathbf{H} = \mathbf{H}^*$$

$$T^2 = +1$$

$$\mathfrak{o}(2M) = \mathfrak{o}(M) \oplus \mathfrak{o}(M) + \mathfrak{m}$$

$$J_7$$

$$\begin{pmatrix} a_1 & \\ & a_2 \end{pmatrix}$$

$$a_1, a_2 \in \mathbb{R} \subset M_8(\mathbb{R})$$

$$\begin{pmatrix} & a_1 \\ a_2 & \end{pmatrix}$$

$$a_1, a_2 \in \mathbb{R} \subset M_8(\mathbb{R})$$

$$\begin{pmatrix} & \mathbf{h}_{pq} \\ \mathbf{h}_{pq}^T & \end{pmatrix}$$

$$\mathbf{h}_{pq} = \mathbf{h}_{pq}^*$$

$$C^2 = +1, T^2 = +1, S^2 = 1$$

$$\mathfrak{o}(M) \oplus \mathfrak{o}(M) = \mathfrak{o}(M) + \mathfrak{m}$$

$$J_8$$

$$\begin{pmatrix} a & \\ & a \end{pmatrix}$$

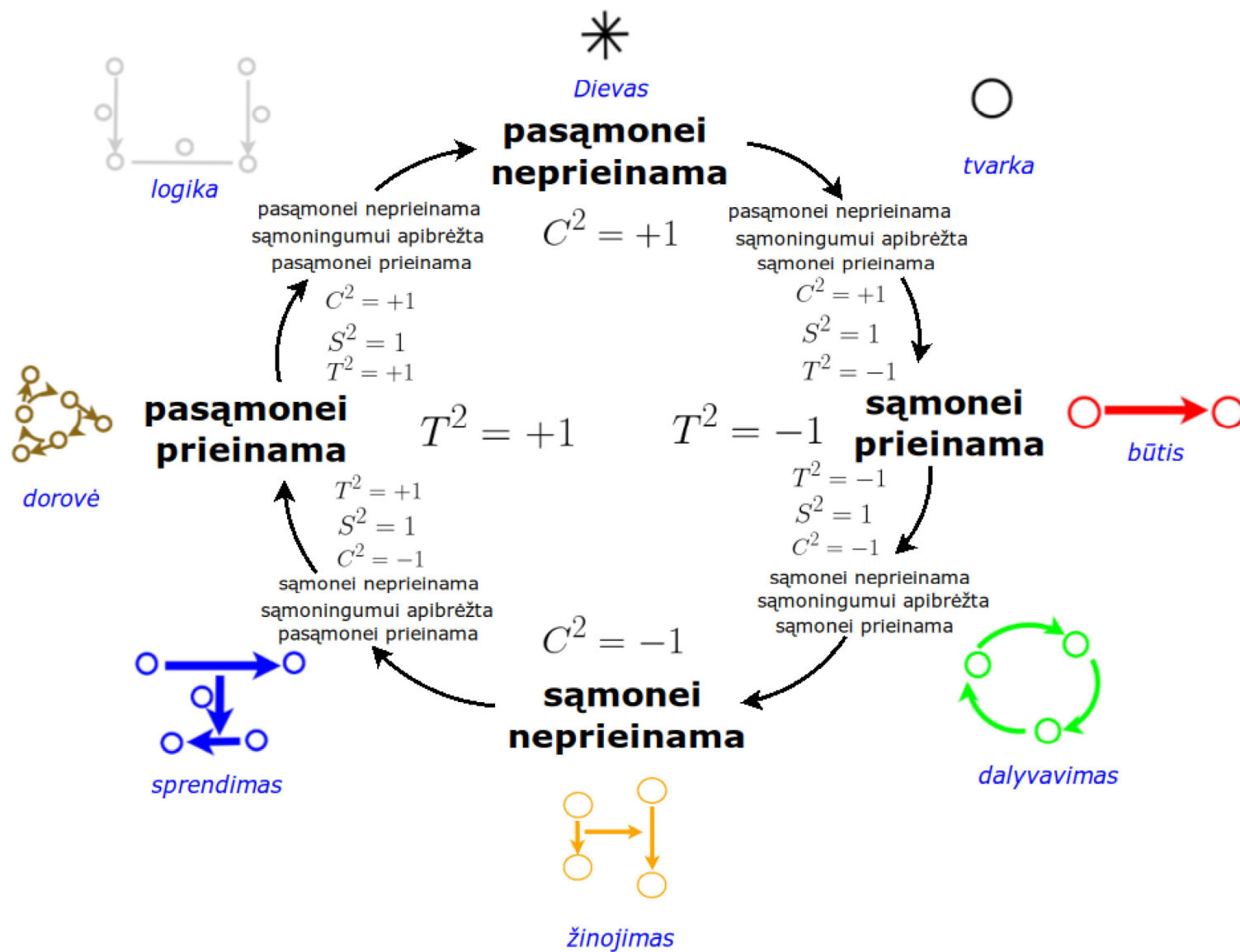
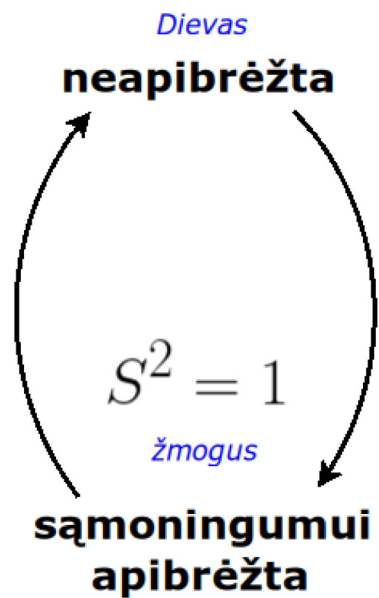
$$a \in \mathbb{R} \subset M_8(\mathbb{R})$$

$$\begin{pmatrix} a & \\ & -a \end{pmatrix}$$

$$a \in \mathbb{R} \subset M_8(\mathbb{R})$$

$$\mathbf{H} = -\mathbf{H}^* = \begin{pmatrix} & -m \\ m & \end{pmatrix}$$

$$C^2 = +1$$



Dėkoju savo draugams už vertingus pokalbius.

Kviečiu prisijungti!

www.Math4Wisdom.com

Andrius Kulikauskas, math4wisdom@gmail.com



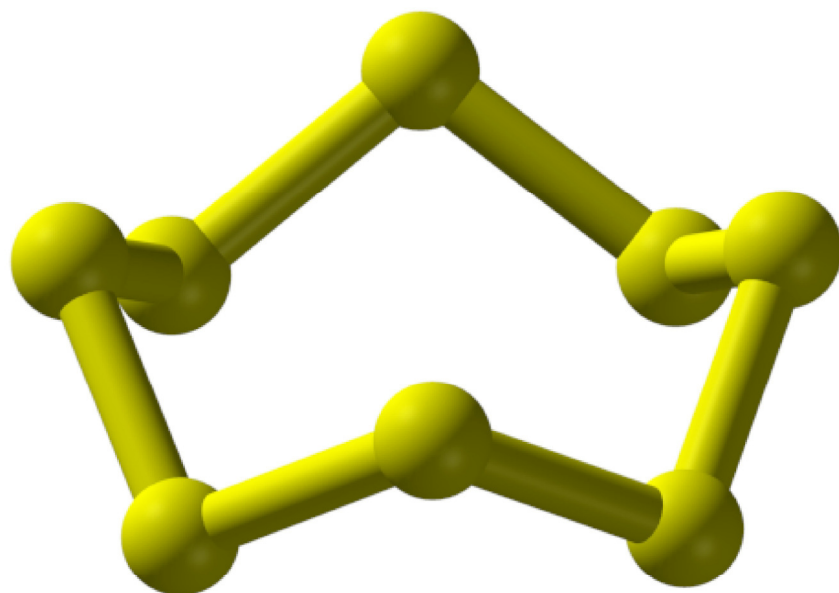
John Harland
Palomar College



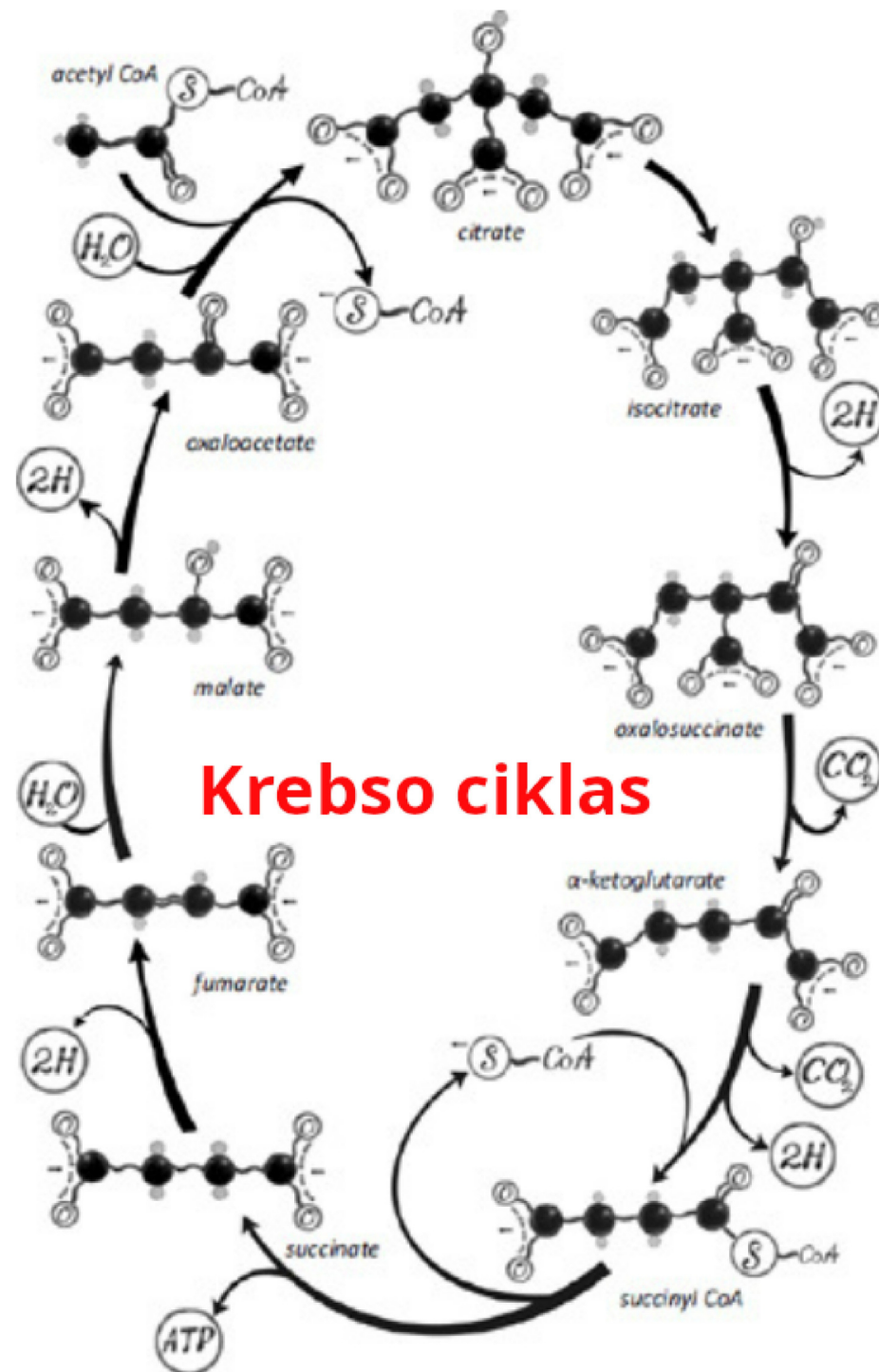
Thomas Gajdosik
Vilniaus universitetas

savasties pavidalas?

S_8 $Z=16$



oktasiera



Nick Lane. Transformer.