HEAP DATA STRUCTURE

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Heap

Heap is a specialized binary tree-based data structure that satisfies the following conditions:

1. A heap is an almost complete binary tree: the first element is stored in the root.

   Then the next level is filled from the left to the right.

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It is easy to show that the highest priority element is always stored at the root.

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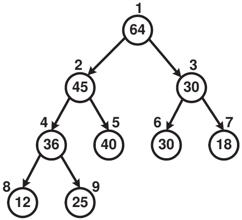
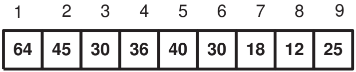
An example of a heap is given in the following figure:



a)



1	2	3	4	5	6	7	8	9
64	45	30	36	40	30	18	12	25

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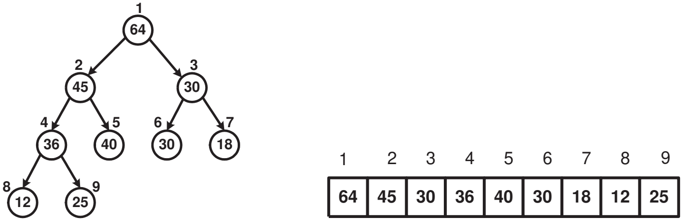

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a) b)

Given a node a_i at index i, its children are at indices $2i$ and $2i + 1$ (elements a_{2i} and a_{2i+1}).

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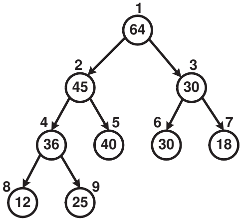

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b)

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A parent of vertex a_i is at index $j = \lfloor i/2 \rfloor$ (element a_j).

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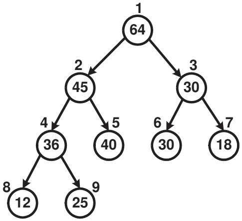

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This simple indexing scheme makes it efficient to move "up" or "down" the binary tree.

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All leaves of the binary tree are already ordered. Such elements are at indices $(\frac{N}{2} + 1), \ldots, N$.

Then we take elements $\frac{N}{2}, \frac{N}{2} - 1, \dots, 1$ and apply recursive checking of the ordering condition.

If this condition is not satisfied sift-down transformation is used to restore heap condition, i.e. the given element is swapped with its largest child.

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This checking step is continued in a new place till heap condition is satisfied (recursion technique).

Heap algorithm

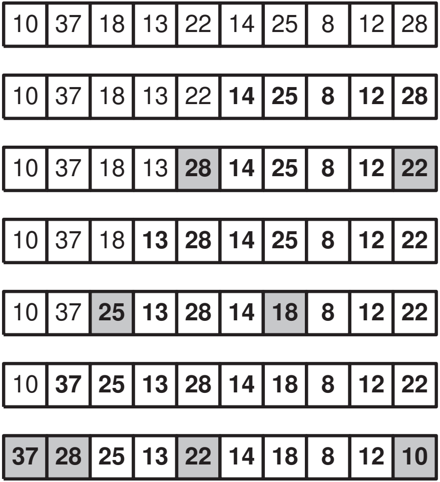
```
MakeHeap ()
begin
    for ( i=1; i $\leqslant$ N; i++ ) do
        $a_i = e_i$;
    end do
    for ( i=N/2; i > 0; i - - ) do
        HeapDownOrder ( i, N );
    end do
end MakeHeap
```

This function is iterative, since at each step only one sub-tree is considered.

```
HeapDownOrder ( p, N )
begin
    i=p;    j = 2i;
    while ( j $\leqslant$ N ) do
        k = j;
        if ( (j+1) $\leqslant$ N ) then
            if ( $a_{j+1} > a_j$ ) k = j+1;
        if ( $a_i < a_k$ ) then
            swap ($a_i$, $a_k$);
            i = k;    j= 2i;
        else
            j = N+1;
end HeapDownOrder
```

Make a heap from array

$$A = (10, 37, 18, 13, 22, 14, 25, 10, 12, 28).$$



10	37	18	13	22	14	25	8	12	28

10	37	18	13	22	**14**	**25**	**8**	**12**	**28**

10	37	18	13	**28**	**14**	**25**	**8**	**12**	**22**

10	37	18	**13**	**28**	**14**	**25**	**8**	**12**	**22**

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37	**28**	**25**	**13**	**22**	**14**	**18**	**8**	**12**	**10**

FIGURE: Construction of heap.

Now we estimate the complexity of this algorithm.

Since a heap is balanced binary tree, its height is equal to $\log N$.

For each activation of HeapDownOrder algorithm the number of comparisons is not larger than $2 \log N$ and the number of swappings is not larger than $\log N$.
Thus total costs can be bounded from above as

$$L(N) \le N \log N, \quad S(N) \le \frac{1}{2} N \log N\,.$$