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# SIMULATION OF TRANSIENTS IN THE MECHANICAL PART OF ELECTROMECHANICAL SYSTEM

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**Abstract.** Electromechanical system as object of research comprises of two parts: electrical and mechanical. Converter of electric energy and control system compose an electrical part, all other linked between them form a mechanical part. Several elements of mechanical parts, for example, shaft can be of finite stiffness and the total system can have kinematical clearance. In the paper two-mass systems with the finite stiffness and non linearity of the shaft caused by kinematical clearance is considered. The structures of electromechanical system with finite stiffness of the shaft and clearance are presented, computer models of those systems are developed and simulation at different controlling inputs and different parameters of the systems is carried out.

**Key words:** electric drive, two-mass system, finite stifness, clearance, model, simulation

## 1. Introduction

Electric drive is an electromechanical system, which performs the conversion of electrical energy to mechanical energy or vice versa for running various processes such as production plants, transportation of people or goods, home appliances, pumps, air compressors, computer disc drives, robots, music or video players. An electrical drive system uses electrical components. These electrical components contain fewer mechanical parts, subjected to wear, then reducing the need to replace these parts, resulting in lower operating costs for the electric drive system. Electric drive systems consist of an electric motor, a transfer mechanism, an electrical energy converter and a control system. The control system consists of a microcontroller with data connection interfaces, data channels (data network), sensors and actuators (motors).

To couple electrical motor with mechanical load, the mechanical drives are used. The basic types of mechanical drives are: a) geared transmission, which provides specific fixed type ratios; b) belt drives, providing flexibility in the positioning of the motor; c) chain drives, providing infinitely variable speeds; d) traction drives, which provides adjustable speed with relatively high speed.

Thus, electromechanical system consists of two parts: electrical and mechanical. Converter of electric energy and control system compose an electrical part, moving masses form a mechanical part (see Fig.1).



Figure 1. Structure of electromechanical system

Rotating parts of the motor and mechanical load are the main components of the mechanical part of drive. Usually they have different parameters of the movement, therefore intermediate mechanical chains such as shafts, reducers, belt-drives and screw-drives as well as clutches are used. Always there is a certain parameter of interest, for example, inertia, speed of rotation, elasticity of mechanical chains, characterizing the movement of these parts or entire system. Two-mass or three-mass systems are characterized with oscillations of controlled parameters. Advanced control methods are used to reduce them, see [1, 2, 3, 4, 6, 8, 9, 10].

The present article discusses the movement of mechanical part which consist of actuator (motor) and mechanical load and dependences of controlled parameters upon elasticity and possible clearance in detail, when torque, developed by the motor is not constant but varies in time by the exponential law.

### 2. Mechanical Part of Electromechanical System

Mechanical part consists of all system chains linked between them mechanically and revolving with different speeds. To simplify solution of the problem usually the simplest case is considered, when electric motor and the load are connected with absolutely stiff shaft and both parts rotate with the same speed. Such a system is called one-mass system. It is characterized by torque, developed by the motor T, mechanical load torque  $T_{12}$ , total inertia J and speed of rotation  $\omega$ . This is the simplest well known model of electromechanical system.

Electromechanical system which is composed of elements with finite stiffness is more complex (Fig.2). The finite stiffness shaft, coupling motor with mechanical load is specific element which appears in the system. In this case instantaneous speeds of the shaft ends are different and the system behavior obtains new features. Electromechanical system, which consists of two masses rotating with different speeds  $\omega_1$  and  $\omega_2$  coupled with the finite stiffness shaft, is called two-mass electromechanical system.



**Figure 2.** Block diagram of two mass mechanical part, i.e. the motor and mechanical load. Here J is inertia,  $\omega_1$  instantaneous rotation speed of motor shaft,  $\omega_2$  instantaneous rotation speed of driven machine shaft, T electromagnetic torque of a motor,  $T_{st}$  load torque,  $M_{12}$  torque of elasticity,  $c_{12}$  stiffness of elastic mechanical part.

Direction of the movement of the drive depends on the acting forces and torques as well as its character depends on properties of electrical circuit and mechanical chains. Several elements of mechanical parts, for example, shaft can have finite stiffness and the total system can have kinematical clearance. According to this, electro-mechanics deals with one-mass, two-mass or severalmass mechanical part of a system.

If any element of the system would have the finite stiffness, for example, a clutch, coupling the motor and mechanical load, then a new element with inertia  $J_2$  and stiffness  $c_{23}$  would appear between the motor and the load in the structure, as presented in Fig.3.



Figure 3. Block diagram of three mass mechanical part.

Obviously, consideration of three-mass system with nonlinearity or clearance is reasonable, just we determine in advance the parameter of movement as a target of interest. In the same way more complex structure can be elaborated if more nonlinear elements of electromechanical system have to be considered.

# 3. Differential Equations and Structure of Two-Mass Mechanical Part

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Movement of mechanical part of electromechanical system in general form is described by the Lagrange equation [5]:

$$\frac{\mathrm{d}}{\mathrm{d}t} \left( \frac{\partial L}{\partial q_i} \right) - \frac{\partial L}{\partial q_i} = Q_i, \tag{3.1}$$

where  $L = W_k - W_p$  is the Lagrange function,  $Q_i$  is a force, depending on elementary works of external forces and their possible displacement  $\partial q_i$ .

In general case, electromechanical systems deal with systems elements of finite stiffness, possible clearance and non-linearity of motor and load characteristics. It is impossible to obtain analytical solutions of this nonlinear differential equations with varying coefficients. To get the results, methods of numerical and computer modeling must be used.

In two-mass system the finite stiffness and non linearity of the shaft caused by kinematical clearance is considered. The system of equations is written in the following way [5, 11]:

$$\begin{cases} T & -T_{st1} - T_{12} = J_1 s \omega_1, \\ & -T_{st2} + T_{12} = J_2 s \omega_2, \\ & s T_{12} = c_{12} (\omega_1 - \omega_2). \end{cases}$$
(3.2)

The block diagram, corresponding to the system of equations (3.2), is shown in Fig.4.



Figure 4. Block diagram of two-mass system.

In dynamic systems a torque of elasticity  $T_{12}$  linearly depends on torsion angle  $\Delta \varphi$  (see Fig.5). This system can be described by the following ystem of equations [5]:

$$T - T_{12} - T_{st1} = J_1 s \omega_1,$$
  

$$T_{12} - T_{st2} = J_2 s \omega_2,$$
  

$$T_{12} = C_{12} \left( \varphi_1 - \varphi_2 - \frac{\Delta \varphi}{2} \right), \text{ if } |\varphi_1 - \varphi_2| < \frac{\Delta \varphi}{2},$$
  

$$T_{12} = 0, \text{ if } |\varphi_1 - \varphi_2| \le \frac{\Delta \varphi}{2}.$$
  
(3.3)

A new element considering kinematical clearance is added to the structure of the system (see Fig.6).



**Figure 5.** Dependence of elasticity torque on torsion angle  $\Delta \varphi$ .



Figure 6. Block diagram of two-mass system with kinematical clearance.

The developed block diagrams give a possibility to obtain the dependences of responses of two-mass system on parameters of the system elements.

## 4. Simulation of Two-Mass System

It is convenient to investigate dynamic characteristics of two-mass system with MATLAB software and SIMULINK. Application of SIMPLORER to SIMULINK interface can be discussed also, but the SIMULINK software [7] was used to develop a model of this system. It gives us a possibility to consider nonlinearities of any type. The model developed for two-mass system without the clearance is given in Fig.7.



Figure 7. Model of two-mass system without clearance.

The presented system requires to be supplemented with the structures of torque T, acting at the system input and load. Electromagnetic torque is the input of the system. Its dependence on time can be described in different ways: by exponential, dead beat and oscillating response. Fig.8 presents one of the possible variants, when torque of the motor changes in time by exponential law.



Figure 8. Dependence of the input torque against time.

SIMULINK model of dynamic system composed from the motor with exponential torque input and fan type load is presented in Fig.9.



Figure 9. SIMULINK model of a dynamic system.

The clearance in this model is expressed by the dead zone. Only electromagnetic transients happen in the system, until the dead zone is not passed. After passing the dead zone, the second mass starts make influence on the dynamics of the system. The response of the system rotation speed and output torque are presented in Fig.10.

The character of speed change can be explained by influence of elastic shaft deformation and clearance. At the beginning of the process, the speed of the motor increases until developed electromagnetic torque becomes equal to the torque of elasticity  $T_{12}$ . The speed of load at this time interval is equal to zero. If the active load is present, this speed can be even negative, i.e. the motor speed changes its direction. When the electromagnetic torque overcomes the torque of elasticity, the motor starts to rotate on no-load, while the clearance still exists. When clearance is passed, motor starts to operate with load and its speed rapidly decreases. At this time the torque, developed by motor, becomes greater and the speed begins to increase and to turn shaft of the load. After some time it reaches steady-state value.

The influence of the finite stiffness is shown in Fig.11.



Figure 10. The response of rotation speed and torque of dynamic system with clearance.



Figure 11. The response of rotation speed at different values of the finite stiffness.

#### 5. Conclusions

- 1. Finite stiffness causes oscillations of rotation speed. Amplitude of oscillations is greater if stiffness of elastic mechanical part is smaller. When stiffness coefficient is equal to infinity, the ideal transient process, corresponding to the dead beat response is obtained.
- 2. The change in speed of the system with clearance at the beginning of starting process corresponds to the rule of no-load electric drive rotation speed while under elastic torque the load appears. During this part of transient process the motor rotation speed increases exponentially.
- 3. Due to clearance the output shaft speed lags behind the motor shaft speed.
- 4. Clearance in two-mass electromechanical system causes dynamical torques.

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