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MATHEMATICAL MODELLING OF MULTIWAVE VOLUME FREE ELECTRON LASER

K. BATRAKOV and S. SYTOVA

Research Institute for Nuclear Problems, Belarusian State University

Bobruiskaya 11, 220050 Minsk, Belarus

E-mail: batrakov@inp.minsk.by; sytova@inp.minsk.by

Abstract. This contribution is devoted to investigation by methods of mathematical modelling of multiwave Volume Free Electron Laser (VFEL). Special emphasis is placed to consideration of three-wave VFEL. The results of mathematical modelling confirmed preliminary estimates. Computer code VOLC for simulation of different schemes of two- and three-wave VFEL is described.

Key words: free electron laser, quasi-Cherenkov instability, simulation, nonlinear integro-differential system

1. Introduction

At present time FELs (Free Electron Lasers) based on different radiation mechanisms are constructed for different wave-length ranges: from centimeter to ultraviolet [6]. Volume Free Electron Laser (VFEL) based on mechanism of multiwave volume distributed feedback (VDFB) was proposed firstly in [3], theoretically investigated in [2] and constructed in 2001 [1]. VDFB allows to reduce significantly starting currents and to tune laser frequency. It provides also mode discrimination in oversized systems (systems with transverse linear sizes essentially exceeding generation wavelength). This gives a possibility of generation in a large volume, distribution of high current beam over large cross-section and reducing of electrical load on laser elements.

This article is devoted to investigation by methods of mathematical modelling of multiwave VFEL. Our previous investigations were devoted to numerical modelling of VFEL with two-wave distributed feedback. Theoretical investigations show the great advantage of different multiwave diffraction geometries and in particular of three-wave diffraction geometry. Generation in multiwave distributed feedback geometry has many advantages including lasing in inaccessible for traditional schemes range of parameters.

2. Brief Review of Basic VFEL Operation Principles

Electron beam with initial electron velocity $\mathbf{u}$ and current density j in VFEL (see Fig. 1) can move close to the target or through the target that is a three-dimensional spatially-periodic structure of the length L. Under diffraction conditions some strong coupled waves are generated. Under proper phase conditions electrons of the beam group in a deceleration phase and produce stimulated emission. In the case of amplification regime external electromagnetic waves are incident to the target. Generator regime can be realized and oscillator regime is realized without external waves.

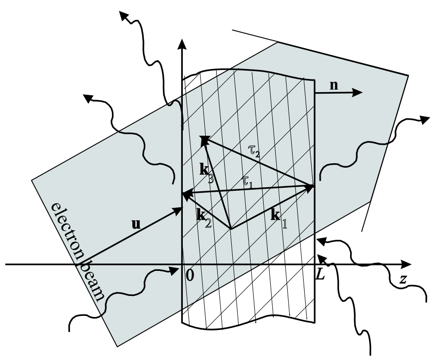


Figure 1. Three-wave VFEL (Bragg-Bragg geometry).

There are three possible geometries in three-wave system. In Bragg-Bragg case, depicted in Fig. 1, we deal with the following geometry:

$$(\mathbf{k}_1, \mathbf{n}) > 0, \quad (\mathbf{k}_2, \mathbf{n}) < 0, \quad (\mathbf{k}_3, \mathbf{n}) < 0,$$

where $\mathbf{n}$ is a normal relative to the surface. Laue-Laue geometry is realized when

$$(\mathbf{k}_1, \mathbf{n}) > 0, \quad (\mathbf{k}_2, \mathbf{n}) > 0, \quad (\mathbf{k}_3, \mathbf{n}) > 0.$$

Bragg-Laue geometry is the case when waves are oriented so that

$$(\mathbf{k}_1, \mathbf{n}) > 0, \quad (\mathbf{k}_2, \mathbf{n}) < 0, \quad (\mathbf{k}_3, \mathbf{n}) > 0.$$

3. Mathematical Model of VFEL

The system of equations for all cases of VFEL is obtained from the Maxwell equations in the slowly-varying envelope approximation.

Mathematical model describing nonlinear processes developing in three-wave VFEL has the following form:

$$\frac{\partial E_1}{\partial t} + a_1 \frac{\partial E_1}{\partial z} + b_{11} E_1 + b_{12} E_2 + b_{13} E_3$$
$$= \Phi \int_0^{2\pi} \frac{2\pi - p}{8\pi^2} \big(\exp\big(-i\Theta(t,z,p) \big) + \exp\big(-i\Theta(t,z,-p) \big) \big)\, dp,$$
$$\frac{\partial E_2}{\partial t} + a_2 \frac{\partial E_2}{\partial z} + b_{21} E_1 + b_{22} E_2 + b_{23} E_3 = 0, \tag{3.1}$$
$$\frac{\partial E_3}{\partial t} + a_3 \frac{\partial E_3}{\partial z} + b_{31} E_1 + b_{32} E_2 + b_{33} E_3 = 0,$$

$$\frac{d^2\Theta(t,z,p)}{dz^2} = \Psi \left(k_1 - \frac{d\Theta(t,z,p)}{dz} \right)^3 \mathrm{Re} \left(E_1(t - \frac{z}{u}, z) \exp(i\Theta(t,z,p) \right), \tag{3.2}$$

$$\Theta(t,0,p) = p, \quad \frac{d\Theta(t,0,p)}{dz} = k_1 - \frac{\omega}{u}, \tag{3.3}$$

$$E_1(t,0) = E_1^0, \quad E_2(t,L_2) = E_2^0, \quad E_3(t,L_3) = E_3^0, \tag{3.4}$$
$$E_i(0,z) = 0, \quad i = 1,2,3,$$

where $t > 0$, $z \in [0, L]$, $p \in [-2\pi, 2\pi]$. This system is a system of integro-differential equations with temporal argument t, spatial coordinate z and initial electron phase p. Amplitudes of electromagnetic field $E_1(t,z)$, $E_2(t,z)$, $E_3(t,z)$ and coefficients a and b are complex-valued. Φ is imaginary. Function $\Theta(t,z,p)$ describes phase of electron beam relative to electromagnetic wave. Θ and coefficient Ψ are real. k_1 is a projection of wave vector $\mathbf{k}_1$ on axis z. ω is a field frequency. Values of boundaries L_2 and L_3 for wave vectors $\mathbf{k}_2$ and $\mathbf{k}_3$ take values 0 or L depending on geometry considered.

Numerical methods to solve the system (3.1)-(3.4) are similar to methods proposed in [4, 5] for simulation of two-wave VFEL.

Let us formulate the system of equations and boundary conditions for common case of n-wave coplanar distributed feedback geometry in the following form:

$$\frac{\partial \mathbf{E}}{\partial t} + \mathbf{A} \frac{\partial \mathbf{E}}{\partial z} + \mathbf{B}\mathbf{E} = \mathbf{G}(I), \tag{3.5}$$

where $\mathbf{E} = (E_i)^T$. Boundary conditions (3.4) are changed to the next form:

$$E_i(t, L_i) = E_i^0, \quad i = 1, \ldots, n. \tag{3.6}$$

Diagonal matrix $\mathbf{A}$ contains direction cosines of wave vectors $\mathbf{k}_i$. Matrix $\mathbf{B}$ describes dynamical diffraction in the system.

4. Computer Code VOLC

Computer code VOLC that means VOLume Code was developed on the basis of multiple Fortran codes, created in 1991–2005 years. Its interface is presented in Fig. 2. This code realizes different geometries of two- and three-wave VFEL. Dimensionality is 2D (one spatial coordinate and one phase space coordinate) plus time. Algorithm of VOLC is presented in Fig. 3. VOLC was tested with carping. There were tested different regimes as oscillator and amplifier regimes, SASE (Self-Amplified Spontaneous Emission), BWT (backward wave tube), TWT (travelling wave tube), BWT-TWT, BWT-BWT-TWT and others. All results correspond to theory prediction.

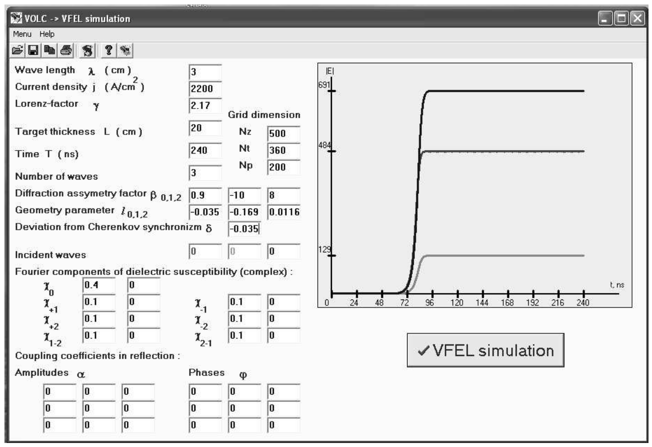


Figure 2. Interface of VOLC.

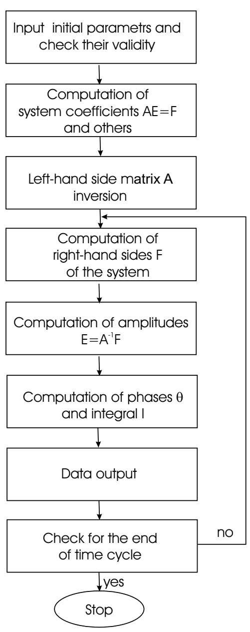
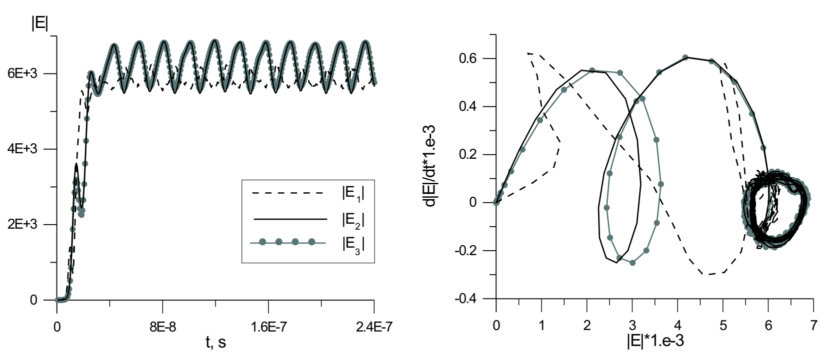


Figure 3. VOLC Algorithm.

5. Results of Numerical Experiments

Some results of numerical experiments carried out using code VOLC are presented here. Different regimes of three-wave VFEL operation were investigated. We obtained different examples of establishment of nonstationary solution including steady-state smooth solutions, oscillations, as well as chaotic regimes. In Fig. 4 the periodic regime of VFEL intensity in Bragg-Bragg geometry and corresponding phase space portrait are given. It is evident that after establishing of periodic regime we deal with periodic 1T and 2T regimes. This is illustrated in Fig. 5 and Fig. 6a but with some computational noise.

One of the important parameters in VFEL operating is a current threshold j_{th}. It is a minimal current density at which the process of generation begins. The possibility to reduce the current threshold is one of the main advantage of VFEL compared to other generator of electromagnetic energy. This was confirmed in numerical experiments carried out. In Fig. 6b the comparison of dependence of current threshold j_{th} on the length of the target L is demonstrated for two- and three-wave geometry. This is a good illustration of effectiveness of volume distributed feedback. It is evident that the threshold current can be significantly decreased in multiwave diffraction geometry.

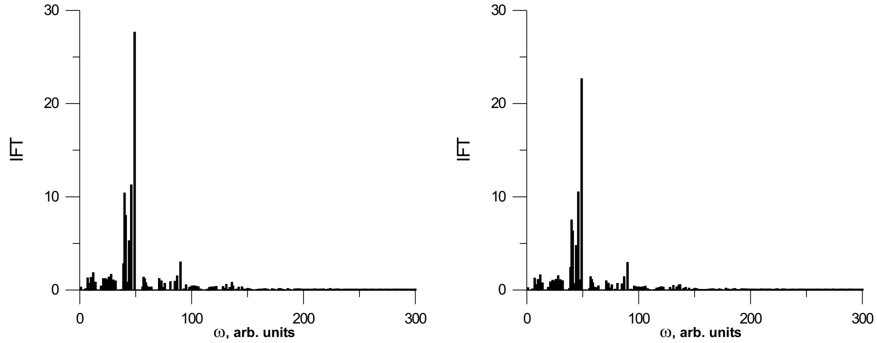


a) b)

Figure 4. Periodic regimes of VFEL intensity (a) and corresponding phase space portrait (b).

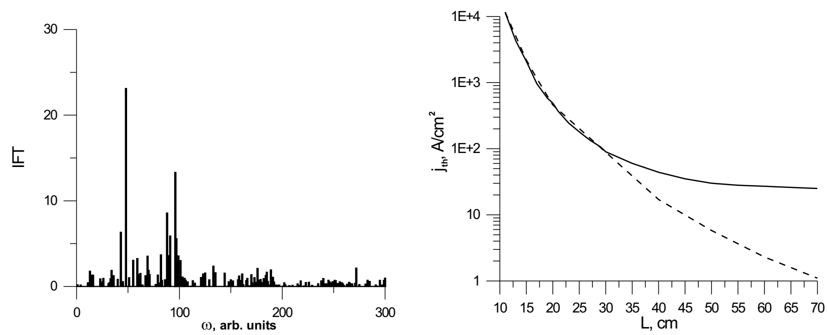
6. Conclusions

Mathematical models and computer code VOLC described here can be used effectively in modelling of nonlinear regimes of VFEL operation. They will be useful for providing experiments on VFEL on the installations created at the Research Institute for Nuclear Problems of Belarusian State University. Authors thank Prof. V. G. Baryshevsky for permanent interest to their work.



a) b)

Figure 5. Fourier 1T periodic regimes corresponding to amplitudes E_2 (a) and E_3 (b) from Fig. 4.



a) b)

Figure 6. (a) Fourier 2T periodic regime corresponding to amplitude E_1 from Fig. 4. (b) Dependence of current threshold j_{th} on length of the target L for two-wave VFEL (solid line) and three-wave VFEL (dashed line).

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